1. [10 points] The following table provides some information on the populations of Detroit and Ann Arbor over time.

<table>
<thead>
<tr>
<th>Year</th>
<th>1970</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann Arbor Population (in thousands)</td>
<td>100</td>
<td>114</td>
</tr>
<tr>
<td>Detroit Population (in thousands)</td>
<td>1514</td>
<td></td>
</tr>
</tbody>
</table>

Remember to show your work clearly.

a. [3 points] Suppose that between 1950 and 2000 the population of Ann Arbor grew at a constant rate (in thousands of people per year). Find a formula for a function $A(t)$ modeling the population of Ann Arbor (in thousands of people) $t$ years after 1950.

Solution: Since the population grew at a constant rate, $A(t)$ is a linear function. The slope of the graph of $A(t)$ is

$$\frac{A(50) - A(20)}{50 - 20} = \frac{114 - 100}{50 - 20} = \frac{14}{30} = \frac{7}{15} \approx 0.467.$$ 

Since $A(20) = 100$, we find that $A(t) = 100 + \frac{7}{15}(t - 20)$.

Answer: $A(t) = 100 + \frac{7}{15}(t - 20)$

b. [5 points] Suppose that between 1950 and 2000, the population of Detroit decreased by 6% every 4 years. Find a formula for an exponential function $D(t)$ modeling the population of Detroit (in thousands of people) $t$ years after 1950.

Solution: Since $D(t)$ is an exponential function, there are constants $a$ and $c$ so that $D(t) = ca^t$. Since the population of Detroit decreased by 6% every four years, $a^4 = 0.94$. Thus $a = (0.94)^{1/4}$ and $D(t) = c(0.94)^{t/4}$

To solve for $c$, we note that

$$1514 = D(20) = c(0.94)^{20/4} = c(0.94)^5$$

and thus

$$c = \frac{1514}{(0.94)^5} \approx 2062.94.$$ 

Answer: $D(t) = \frac{1514}{(0.94)^5}(0.94)^{t/4}$ or $1514(0.94)^{(t-20)/4}$

c. [2 points] According to your model $D(t)$, what was the population of Detroit in the year 2000? Include units.

Solution: $D(50) = \frac{1514}{(0.94)^5}(0.94)^{50/4} \approx 951.89$.

Answer: about 952 thousand people