1. [10 points] The following table provides some information on the populations of Detroit and Ann Arbor over time.

Year	1970	2000
Ann Arbor Population (in thousands)	100	114
Detroit Population (in thousands)	1514	

Remember to show your work clearly.

a. [3 points] Suppose that between 1950 and 2000 the population of Ann Arbor grew at a constant rate (in thousands of people per year). Find a formula for a function $A(t)$ modeling the population of Ann Arbor (in thousands of people) t years after 1950.

Solution: Since the population grew at a constant rate, $A(t)$ is a linear function. The slope of the graph of $A(t)$ is

$$\frac{A(50)-A(20)}{50-20} = \frac{114-100}{50-20} = \frac{14}{30} = \frac{7}{15} \approx 0.467\,.$$

Since $A(20) = 100$, we find that $A(t) = 100 + \frac{7}{15}(t-20)$.

Answer: $A(t) =$ $100 + \frac{7}{15}(t-20) = \frac{272}{3} + \frac{7}{15}t \approx 90.67 + 0.467t$

b. [5 points] Suppose that between 1950 and 2000, the population of Detroit decreased by 6% every 4 years. Find a formula for an exponential function $D(t)$ modeling the population of Detroit (in thousands of people) t years after 1950.

Solution: Since $D(t)$ is an exponential function, there are constants a and c so that $D(t) = ca^t$. Since the population of Detroit decreased by 6% every four years, $a^4 = 0.94$. Thus $a = (0.94)^{1/4}$ and $D(t) = c(0.94)^{t/4}$
To solve for c, we note that

$$1514 = D(20) = c(0.94)^{20/4} = c(0.94)^5$$

and thus

$$c = \frac{1514}{(0.94)^5} \approx 2062.94\,.$$

Answer: $D(t) =$ $\frac{1514}{(0.94)^5}(0.94)^{t/4}$ **or** $1514(0.94)^{(t-20)/4}$

c. [2 points] According to your model $D(t)$, what was the population of Detroit in the year 2000? *Include units.*

Solution: $D(50) = \frac{1514}{(0.94)^5}(0.94)^{50/4} \approx 951.89.$

Answer: **about 952 thousand people**