3. [9 points] Consider the function h defined by

$$h(x)=\begin{cases}\dfrac{60(x^2-x)}{(x^2+1)(3-x)} & \text{for } x<2\\ c & \text{for } x=2\\ 5e^{ax}-1 & \text{for } x>2\end{cases}$$

where a and c are constants.

a. [5 points] Find values of a and c so that both of the following conditions hold.

- $\lim_{x\to 2} h(x)$ exists.
- $h(x)$ is not continuous at $x=2$.

Note that this problem may have more than one correct answer. You only need to find one value of a and one value of c so that both conditions above hold. Remember to show your work clearly.

Solution: In order for $\lim_{x\to 2} h(x)$ to exist, it must be true that $\lim_{x\to 2^-} h(x) = \lim_{x\to 2^+} h(x)$. Now $\lim_{x\to 2^-} h(x) = \dfrac{60(2^2-2)}{(2^2+1)(3-2)} = 24$ and $\lim_{x\to 2^+} h(x) = 5e^{2a}-1$. So it follows that $5e^{2a}-1=24$. Solving for a, we have

$$\begin{aligned}5e^{2a}-1&=24\\ e^{2a}&=5\\ 2a&=\ln(5)\\ a&=\ln(5)/2\approx 0.804.\end{aligned}$$

When $a=\ln(5)/2$, $\lim_{x\to 2} h(x) = 5e^{(\ln(5)/2)*2} = 5e^{\ln(5)}-1=24$. So, h is not continuous at $x=2$ as long as $\lim_{x\to 2} h(x)\neq h(2)$. Since $h(2)=c$, we can choose c to be any number other than 24.

Answer: $a =$ $\ln(5)/2$ and $c =$ **7 (or any value other than 24)**

b. [2 points] Determine $\lim_{x\to -\infty} h(x)$. If the limit does not exist, write DNE.

Solution: By looking at the rational function

$$\frac{60(x^2-x)}{(x^2+1)(3-x)} = \frac{60(x^2-x)}{-x^3+3x^2-x+3},$$

(the relevant piece of the function here) we see that as $x\to-\infty$, $h(x)$ approaches 0.

Answer: $\lim_{x\to -\infty} h(x) =$ 0

c. [2 points] Find all vertical asymptotes of the graph of $h(x)$. If there are none, write None.

Answer: Vertical asymptote(s): None