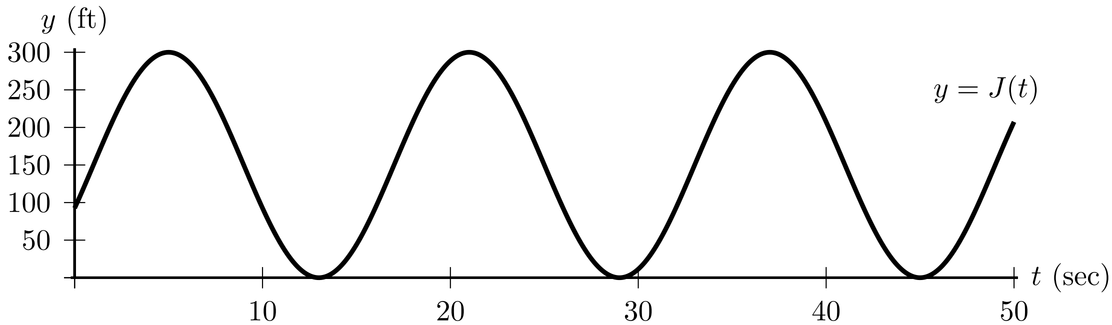
4. [12 points] A dare devil jumps off the side of a bungee jumping platform while attached to a magically elastic bungee cord. Just a few moments after the jump begins, a timer is started and her position is recorded. At t seconds after the timer begins, her distance in feet below the platform is given by the function

$$J(t) = -150\cos(0.125\pi(t+3)) + 150\,.$$

A portion of the graph of $y = J(t)$ is shown below.



Throughout this problem, do not make estimates using the graph.

a. [2 points] Compute the average velocity of the bungee jumper during the first 16 seconds after the timer begins.

Solution: Since $0.125\pi = 2\pi/(\text{period of } J(t))$, the period of $J(t)$ is $2\pi/(0.125\pi) = 16$. Thus, $J(0) = J(16)$ and

$$\text{average velocity} = \frac{J(16) - J(0)}{16} = \frac{0}{16} = 0 \text{ ft/s}\,.$$

Answer: average velocity = 0 **ft/s**

b. [3 points] Recall that *average speed* over an interval of time is given by $\frac{\text{distance traveled}}{\text{time elapsed}}$. Compute the average speed of the bungee jumper during the first 16 seconds after the timer begins.

Solution: Since $J(t)$ has period 16 and amplitude 150,

$$\text{distance traveled during the interval } 0 \le t \le 16 = 4(\text{amplitude}) = 600 \text{ ft}$$

Thus, the average speed is $(600 \text{ ft})/(16 \text{ s}) = 37.5$ ft/s.

Answer: average speed = 37.5 **ft/s**

c. [5 points] Use the limit definition of instantaneous velocity to write an explicit expression for the instantaneous velocity of the bungee jumper 2 seconds after the timer begins. *Your answer should not involve the letter J. Do not attempt to evaluate or simplify the limit.*

Answer: $\lim\limits_{h\to 0} \dfrac{-150\cos(0.125\pi(5+h)) + 150 - (-150\cos(0.125\pi(5)) + 150)}{h}$

d. [2 points] Find all values of t in the interval $0 \le t \le 30$ when the instantaneous velocity of the bungee jumper is 0 feet per second.

Solution: The instantaneous velocity is 0 when the tangent line to the position graph is horizontal. This occurs at the maxima and minima on the sinusoidal graph. The first maximum occurs at $t = 5$ and so the first minimum occurs at $t = 13$ (half a period later).

Answer: $5, 13, 21, 29$