

5. [11 points] Oren, a Math 115 student, realizes that the more caffeine he consumes, the faster he completes his online homework assignments. Before starting tonight's assignment, he buys a cup of coffee containing a total of 100 milligrams of caffeine.

Let  $T(c)$  be the number of minutes it will take Oren to complete tonight's assignment if he consumes  $c$  milligrams of caffeine. Suppose that  $T$  is continuous and differentiable.

- a. [2 points] Circle the ONE sentence below that is best supported by the statement "the more caffeine he consumes, the faster he completes his online homework assignments."

i.  $T'(c) \geq 0$  for every value  $c$  in the domain of  $T$ .

ii.  $T'(c) \leq 0$  for every value  $c$  in the domain of  $T$ .

iii.  $T'(c) = 0$  for every value  $c$  in the domain of  $T$ .

- b. [1 point] Explain, in the context of this problem, why it is reasonable to assume that  $T(c)$  is invertible.

*Solution:* Since the more caffeine Oren consumes the faster he is able to finish his homework,  $T(c)$  is a decreasing function. Thus,  $T(c)$  is invertible.

- c. [2 points] Interpret the equation  $T^{-1}(100) = 45$  in the context of this problem. Use a complete sentence and include units.

*Solution:* In order for Oren to complete his homework assignment in 100 minutes, he must consume 45 milligrams of caffeine.

- d. [3 points] Suppose that  $p$  and  $k$  are constants. In the equation  $T'(p) = k$ , what are the units on  $p$  and  $k$ ?

**Answer:** Units on  $p$  are milligrams of caffeine

**Answer:** Units on  $k$  are minutes per milligram of caffeine

- e. [3 points] Which of the statements below is best supported by the equation  $(T^{-1})'(20) = -10$ ? Circle the ONE best answer.

i. If Oren has consumed 20 milligrams of caffeine, then consuming an additional milligram of caffeine will save him about 10 minutes on tonight's assignment.

ii. The amount of caffeine that will result in Oren finishing his homework in 21 minutes is approximately 10 milligrams greater than the amount of caffeine that Oren will need in order to finish his homework in 20 minutes.

iii. The rate at which Oren is consuming caffeine 20 minutes into his homework assignment is decreasing by 10 milligrams per minute.

iv. In order to complete tonight's assignment in 19 rather than 20 minutes, Oren needs to consume about 10 milligrams of additional caffeine.

v. If Oren consumes 20 milligrams of caffeine, then he will finish tonight's assignment approximately 10 minutes faster than if he consumes no caffeine.