8. [6 points] Given below is the graph of a sinusoidal function R(x).



Find a possible formula for R(x).

Solution: The graph shown above is of a sinusoidal function with amplitude 4, period 3, and midline y = -2. We first consider the graph of

$$y = 4\cos\left(\frac{2\pi}{3}x\right) - 2\,.$$

This graph has the proper amplitude, period, and midline. We shift this graph over to the right 1 unit to obtain the graph of y = R(x). Thus, one possible formula for R(x) is given by

$$R(x) = 4\cos\left(\frac{2\pi}{3}(x-1)\right) - 2.$$

Answer:
$$R(x) =$$
 $\frac{4\cos\left(\frac{2\pi}{3}(x-1)\right) - 2}{2}$

9. [4 points] The table below gives several values of a function w(x).

x	4.5	4.9	4.99	5	5.01	5.1	5.5
w(x)	-0.879	-0.154	-0.015	0	0.060	0.630	3.750

Use the information in the table above to estimate the following limit.

$$\lim_{h \to 0^-} \frac{w(5+h)}{h}$$

Clearly show any computations that you use to make this estimate.

Solution: The left limit can be approximated by w(5+h)/h for small negative values of h. The table of values provided for w allows us to compute this when h = -0.1 and when h = -0.01. The results are shown in the table below.

$$\begin{array}{c|c}
h & \frac{w(5+h)}{h} \\
\hline
-0.1 & \frac{w(4.9)}{-0.1} = 1.54 \\
\hline
-0.01 & \frac{w(4.99)}{-.01} = 1.5
\end{array}$$

Using these values, we estimate that the desired left-hand limit is approximately 1.5.

Answer:
$$\lim_{h \to 0^{-}} \frac{w(5+h)}{h} \approx _____1.5$$