2. [12 points] Angelica Neiring and Simona Koloji decide to enjoy the fall weather by racing each other from the brass block "M" in the center of the Diag along a 2.5 kilometer (2500 meter) route to the Huron River inside the Arb. Let $A(t)$ (respectively $S(t)$) be Angelica's (respectively Simona's) distance along the route (in meters) t seconds after they start racing. Angelica and Simona are both wearing GPS watches that record data about their race. The table of values for the functions A and S below shows some of the resulting data, rounded to the nearest meter. Note that the data is not always recorded at regular intervals.

t	0	30	60	66	72	105	114	120	135	168	180	198	300
$A(t)$	0	55	119	137	156	226	249	265	302	384	415	463	737
$S(t)$	0	57	120	137	156	225	248	264	303	389	422	473	768

Use the data above to answer the questions below. Remember to show your work.

a. [2 points] Estimate Angelica's instantaneous velocity 3 minutes into the race. *Include units.*

Solution: We estimate using average velocity based on nearby measurements:

Avg velocity between 168 and 180 seconds: $\dfrac{A(180)-A(168)}{180-168} = \dfrac{415-384}{12} = \dfrac{31}{12} \approx 2.58 \text{ m/sec}$

Avg velocity between 180 and 198 seconds: $\dfrac{A(198)-A(180)}{198-180} = \dfrac{463-415}{18} = \dfrac{48}{18} \approx 2.67 \text{ m/sec}$

So we estimate that Angelica's instantaneous velocity 3 minutes into the race was about 2.6 meters per second. (Any estimate between the average velocities computed above would be reasonable. We rounded to two significant digits.)

Answer: **About 2.6 meters per second**

b. [2 points] Estimate $S'(120)$.

Solution: We estimate the derivative using nearby measurements:

Average rate of change of $S(t)$ for $114 \le t \le 120$: $\dfrac{S(120)-S(114)}{120-114} = \dfrac{264-248}{6} = \dfrac{16}{6} \approx 2.67$

Average rate of change of $S(t)$ for $120 \le t \le 135$: $\dfrac{S(135)-S(120)}{135-120} = \dfrac{303-264}{15} = \dfrac{39}{15} \approx 2.6$

So we estimate that $S'(120) \approx 2.6$. (Any estimate between the average rates of change computed above would be reasonable. We rounded to two significant digits here.)

Answer: $S'(120) \approx 2.6$

c. [2 points] In the context of this problem, what are the units on the quantity $(A^{-1})'(150)$?

Answer: **seconds per meter**

For questions **d.** and **e.** below, circle the one best answer or circle CANNOT BE DETERMINED if there is not enough information to definitely determine the answer. You do not need to show your work or provide justification for your answers for these questions.

d. [1 point] Who was ahead 5 minutes into the race?

Angelica [✓ Simona] CANNOT BE DETERMINED

e. [1 point] Who was running faster exactly one minute into the race?

Angelica Simona [✓ CANNOT BE DETERMINED]

f. [4 points] In describing the race later, Simona says that her average velocity during the entire race was 2.8 meters per second while Angelica says that after the first 5 minutes, her average velocity for the rest of the race was 3.1 meters per second.
Assuming their statements and the table of values above are accurate, who won the race? Or is there not enough information to decide? Explain your reasoning.

Answer: (Circle one.) [✓ Angelica] Simona Not enough information

Explanation:

Solution: With an average velocity of 2.8 meters per second for the whole race of 2500 meters, it took Simona $\dfrac{2500}{2.8} \approx 892.9$ seconds to complete the 2500 meter race. On the other hand, it took Angelica 300 seconds to run the first 737 meters and, with an average velocity of 3.1 meters per second for the rest of the race $(2500 - 737$ meters), it took her an additional $\frac{2500-737}{3.1} \approx 568.7$ seconds to finish the race. So Angelica's total time for the race was about $300 + 568.7 = 867.7$ seconds, which is less than Simona's time of about 892.9 seconds. Hence Angelica won the race.