

5. [8 points] *Remember to show your work carefully throughout this problem.*

Algie and Cal go on a picnic, arriving at 12:00 noon.

- a. [5 points] Five minutes after they arrive, they notice that 5 ants have joined their picnic. More ants soon appear, and after careful study, they determine that the number of ants appears to be increasing by 20% every minute. Find a formula for a function $A(t)$ modeling the number of ants present at the picnic t minutes past noon for $t \geq 5$.

Solution: Since this is an exponential function, there are constants c and b such that $A(t) = cb^t$. We can see immediately that $b = 1.2$. We can then use the fact that we know that $A(5) = 5$ to find c : $c(1.2)^5 = 5$, so $c = 5/(1.2)^5$, which is approximately 2.01. Alternatively, we can use a horizontal shift to say that this is $5(1.2)^{t-5}$.

Answer: $A(t) = \underline{5(1.2)^{(t-5)} = \frac{5}{1.2^5}(1.2)^t}$

- b. [3 points] Algie and Cal notice that their food is, unfortunately, also attracting flies. The number of flies at their picnic t minutes after noon can be modeled by the function $g(t) = 1.8(1.25)^t$. Algie and Cal decide they will end their picnic when there are at least 1000 flies. How long will their picnic last? *Include units.*

Solution: We wish to find t such that $1.8(1.25)^t = 1000$. Then

$$1.8(1.25)^t = 1000$$

$$\ln(1.8(1.25)^t) = \ln(1000)$$

$$\ln(1.8) + t \ln(1.25) = \ln(1000)$$

$$t \ln(1.25) = \ln(1000) - \ln(1.8)$$

$$t = \ln(1000/1.8)/\ln(1.25) \approx 28.3.$$

So they end their picnic about 28.3 minutes after noon (when it started).

Answer: $\underline{\text{About 28.3 minutes}}$

6. [6 points] Consider the function

$$R(w) = 2 + (\ln(w))^{\cos(w)}.$$

Use the limit definition of the derivative to write an explicit expression for $R'(\pi)$.

Your answer should not involve the letter R . Do not attempt to evaluate or simplify the limit.

Please write your final answer in the answer box provided below.

Answer: $R'(\pi) = \boxed{\lim_{h \rightarrow 0} \frac{(2 + (\ln(\pi + h))^{\cos(\pi + h)}) - (2 + (\ln(\pi))^{\cos(\pi)})}{h}}$