7. [12 points] Phillip Asafy and Genevieve Omicks both enjoy hot chocolate when it's cool outside. They made a few measurements, and these appear in the table below.

P (respectively G) is Phil's (respectively Gen's) consumption of hot chocolate (in quarts, measured to the nearest tenth of a quart) in a month when the average daily high temperature is H (in degrees Celsius, measured to the nearest degree).

$H(°C)$	P (quarts)	G (quarts)
3	16.1	13.3
7	12.8	11.6
15	8.0	6.5

a. [8 points] Based on this data, could either student's monthly hot chocolate consumption be reasonably modeled as a linear function of average daily high temperature? An exponential function? Neither? Carefully justify your answer in the space below.
(Hint: *At least* one of these can be modeled by a linear or an exponential function!)

Solution: First consider Phil's hot chocolate consumption. Suppose $P = p(H)$. To check whether $p(H)$ could be modeled by a linear function, we compute the average rate of change of p over the intervals $[3, 7]$ and $[7, 15]$. We have

$$\frac{p(7)-p(3)}{7-3} = \frac{12.8-16.1}{4} = -0.825 \quad \text{and} \quad \frac{p(15)-p(7)}{15-7} = \frac{8.0-12.8}{8} = -0.6.$$

Since these two average rates of change are quite different, Phil's hot chocolate consumption is not reasonably modeled by a linear function. To check whether $p(H)$ could be modeled by an exponential function, we compute the percent rate of change of $p(H)$ over the intervals $[3, 7]$ and $[7, 15]$. We have

$$\left(\frac{p(7)}{p(3)}\right)^{\frac{1}{7-3}} = \left(\frac{12.8}{16.1}\right)^{\frac{1}{4}} \approx 0.9443 \quad \text{and} \quad \left(\frac{p(15)}{p(7)}\right)^{\frac{1}{15-7}} = \left(\frac{8.0}{12.8}\right)^{\frac{1}{8}} \approx 0.9429.$$

The difference between these percent rates of change is less than 0.2%, so based on this data, $p(H)$ can be reasonably modeled by an exponential function. In particular, we can check that we obtain the data in the table for P using, for example, $19.2(0.9435)^H$
Now consider Gen's hot chocolate consumption. Suppose $G = g(H)$. From the calculations

$$\frac{g(7)-g(3)}{7-3} = \frac{11.6-13.3}{4} = -0.425 \quad \text{and} \quad \frac{g(15)-g(7)}{15-7} = \frac{6.5-11.6}{8} = -0.6375$$

we conclude that Gen's hot chocolate consumption is not reasonably modeled by a linear function. From the calculations

$$\left(\frac{g(7)}{g(3)}\right)^{\frac{1}{7-3}} = \left(\frac{11.6}{13.3}\right)^{\frac{1}{4}} \approx 0.9664 \quad \text{and} \quad \left(\frac{g(15)}{g(7)}\right)^{\frac{1}{15-7}} = \left(\frac{6.5}{11.6}\right)^{\frac{1}{8}} \approx 0.9302$$

we conclude that Gen's hot chocolate consumption can't be reasonably modeled by an exponential function.
(Note that for the exponential cases we could instead compare, for example, $\left(\frac{p(7)}{p(3)}\right)^2$ with $\frac{p(15)}{p(7)}$.)

Answers: Circle one choice for each student.

Phil's consumption P: linear **[exponential]** neither linear nor exponential

Gen's consumption G: linear exponential **[neither linear nor exponential]**

b. [4 points] For this investigation, their friend Maddy measures temperature in degrees Fahrenheit, and she measures her hot chocolate consumption in cups. She finds a function $M(f)$ which is the number of cups of hot chocolate she consumes in a month when the average daily high temperature is f degrees Fahrenheit. If $Q(H)$ is the number of *quarts* of hot chocolate Maddy consumes when the average monthly temperature is H degrees *Celsius*, write a formula for $Q(H)$ in terms of M and H.
Recall that there are 4 cups in a quart and that the conversion from Fahrenheit to Celsius is given by $y = \frac{5}{9}(x - 32)$ *(where* $y^\circ C$ *and* $x^\circ F$ *describe the same temperature).*

Solution: H degrees Celsius is the same as $\frac{9}{5}H + 32$ degrees Fahrenheit, and $M\left(\frac{9}{5}H + 32\right)$ gives Maddy's hot chocolate consumption in cups. We divide this quantity by 4 to convert from cups to quarts.

Answer: $Q(H) = \dfrac{M\left(\frac{9}{5}H + 32\right)}{4}$