7. [12 points] Phillip Asafy and Genevieve Omicks both enjoy hot chocolate when it’s cool outside. They made a few measurements, and these appear in the table below.

<table>
<thead>
<tr>
<th>(H(°C))</th>
<th>(P) (quarts)</th>
<th>(G) (quarts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>16.1</td>
<td>13.3</td>
</tr>
<tr>
<td>7</td>
<td>12.8</td>
<td>11.6</td>
</tr>
<tr>
<td>15</td>
<td>8.0</td>
<td>6.5</td>
</tr>
</tbody>
</table>

\(P\) (respectively \(G\)) is Phil’s (respectively Gen’s) consumption of hot chocolate (in quarts, measured to the nearest tenth of a quart) in a month when the average daily high temperature is \(H\) (in degrees Celsius, measured to the nearest degree).

a. [8 points] Based on this data, could either student’s monthly hot chocolate consumption be reasonably modeled as a linear function of average daily high temperature? An exponential function? Neither? Carefully justify your answer in the space below. (Hint: At least one of these can be modeled by a linear or an exponential function!)

**Solution:**
First consider Phil’s hot chocolate consumption. Suppose \(P = p(H)\). To check whether \(p(H)\) could be modeled by a linear function, we compute the average rate of change of \(p\) over the intervals \([3, 7]\) and \([7, 15]\). We have

\[
\frac{p(7) - p(3)}{7 - 3} = \frac{12.8 - 16.1}{4} = -0.825 \quad \text{and} \quad \frac{p(15) - p(7)}{15 - 7} = \frac{8.0 - 12.8}{8} = -0.6.
\]

Since these two average rates of change are quite different, Phil’s hot chocolate consumption is not reasonably modeled by a linear function.

To check whether \(p(H)\) could be modeled by an exponential function, we compute the percent rate of change of \(p(H)\) over the intervals \([3, 7]\) and \([7, 15]\). We have

\[
\left(\frac{p(7)}{p(3)}\right)^{1/4} = \left(\frac{12.8}{16.1}\right)^{1/4} \approx 0.9443 \quad \text{and} \quad \left(\frac{p(15)}{p(7)}\right)^{1/8} = \left(\frac{8.0}{12.8}\right)^{1/8} \approx 0.9429.
\]

The difference between these percent rates of change is less than 0.2%, so based on this data, \(p(H)\) can be reasonably modeled by an exponential function. In particular, we can check that we obtain the data in the table for \(P\) using, for example, \(19.2(0.9435)^H\).

Now consider Gen’s hot chocolate consumption. Suppose \(G = g(H)\). From the calculations

\[
\frac{g(7) - g(3)}{7 - 3} = \frac{11.6 - 13.3}{4} = -0.425 \quad \text{and} \quad \frac{g(15) - g(7)}{15 - 7} = \frac{6.5 - 11.6}{8} = -0.6375
\]

we conclude that Gen’s hot chocolate consumption is not reasonably modeled by a linear function.

From the calculations

\[
\left(\frac{g(7)}{g(3)}\right)^{1/4} = \left(\frac{11.6}{13.3}\right)^{1/4} \approx 0.9664 \quad \text{and} \quad \left(\frac{g(15)}{g(7)}\right)^{1/8} = \left(\frac{6.5}{11.6}\right)^{1/8} \approx 0.9302
\]

we conclude that Gen’s hot chocolate consumption can’t be reasonably modeled by an exponential function.

(Note that for the exponential cases we could instead compare, for example, \(\left(\frac{p(7)}{p(3)}\right)^2\) with \(\frac{p(15)}{p(7)}\).)

**Answers:** Circle one choice for each student.

Phil’s consumption \(P\): linear [ ] exponential [ ] neither linear nor exponential [ ]

Gen’s consumption \(G\): linear [ ] exponential [ ] neither linear nor exponential [ ]
b. [4 points] For this investigation, their friend Maddy measures temperature in degrees Fahrenheit, and she measures her hot chocolate consumption in cups. She finds a function $M(f)$ which is the number of cups of hot chocolate she consumes in a month when the average daily high temperature is $f$ degrees Fahrenheit. If $Q(H)$ is the number of quarts of hot chocolate Maddy consumes when the average monthly temperature is $H$ degrees Celsius, write a formula for $Q(H)$ in terms of $M$ and $H$.

Recall that there are 4 cups in a quart and that the conversion from Fahrenheit to Celsius is given by $y = \frac{5}{9}(x - 32)$ (where $y^\circ C$ and $x^\circ F$ describe the same temperature).

Solution: $H$ degrees Celsius is the same as $\frac{9}{5}H + 32$ degrees Fahrenheit, and $M\left(\frac{9}{5}H + 32\right)$ gives Maddy’s hot chocolate consumption in cups. We divide this quantity by 4 to convert from cups to quarts.

Answer: $Q(H) = \frac{M\left(\frac{9}{5}H + 32\right)}{4}$