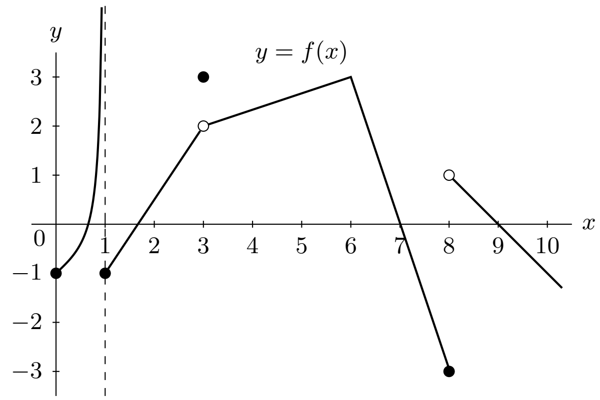
8. [12 points] A portion of the graph of a function f is shown below.



a. [2 points] Give all values c in the interval $0 < c < 10$ for which $\lim_{x\to c} f(x)$ does not exist. If there are none, write NONE.

Answer: $c =$ 1, 8

b. [2 points] Give all values c in the interval $0 < c < 10$ for which $\lim_{x\to c^+} f(x)$ does not exist. If there are none, write NONE.

Answer: $c =$ **NONE**

c. [2 points] Give all values c in the interval $0 < c < 10$ for which $f(x)$ is not continuous at c. If there are none, write NONE.

Answer: $c =$ 1, 3, 8

d. [6 points] With f as shown in the graph above, define a function g by the formula

$$g(x) = \begin{cases} \dfrac{B + 2x^2 + 3x^3 + Ax^5}{12 + 6x^3 + 4x^5} & \text{if } x \le 0 \\ f(x) & \text{if } 0 < x < 10 \end{cases}$$

where A and B are nonzero constants.
Find values of A and B so that both of the following conditions hold.

- $g(x)$ is continuous at $x = 0$.
- $\lim_{x\to-\infty} g(x) = \dfrac{1}{2}$.

If no such values exist, write NONE in the answer blanks.
Be sure to show your work or explain your reasoning.

Solution: To satisfy the first condition, we first compute $g(0)$ by plugging in $x = 0$ to the rational function $\dfrac{B + 2x^2 + 3x^3 + Ax^5}{12 + 6x^3 + 4x^5}$ to find $\lim_{x\to 0-} g(x) = g(0) = \dfrac{B}{12}$. In order for $g(x)$ to be continuous at $x = 0$, we must also have $\lim_{x\to 0^+} g(x) = \dfrac{B}{12}$.
Now $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} f(x) = -1$ (from the graph), so $\frac{B}{12} = -1$, and $B = -12$.
To satisfy the second condition, we compute that

$$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} \frac{B + 2x^2 + 3x^3 + Ax^5}{12 + 6x^3 + 4x^5} = \frac{A}{4}.$$

In order for this limit to equal $\frac{1}{2}$, we must have $A/4 = 1/2$, so $A = 2$.

Answer: $A =$ 2 and $B =$ -12