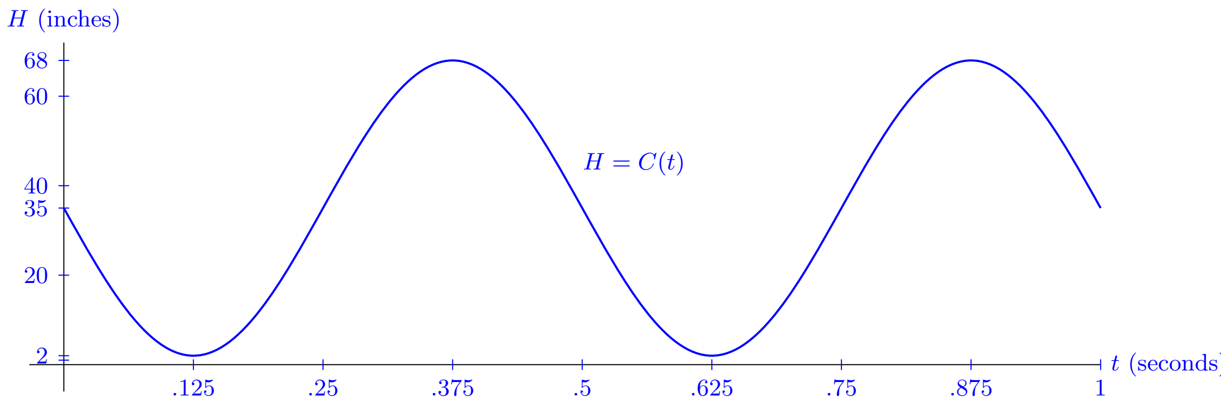
9. [13 points] Cal is jumping a rope being swung by Gen and Algie while Maddy runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Maddy starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $C(t)$, where t is the number of seconds displayed on Maddy's stopwatch.

a. [4 points] On the axes provided below, sketch a well-labeled graph of two periods of $C(t)$ beginning at $t = 0$.
Pay attention to both the shape of your graph and the location of important points.



b. [4 points] Find a formula for $C(t)$.

Answer: $C(t) =$ $35 - 33\sin(4\pi t)$

c. [5 points] Now Gen takes a turn at jumping while Cal and Algie swing the rope. Maddy resets the stopwatch and starts it over again. Let $G(w)$ be the height (in inches above the ground) of the piece of tape when Maddy's stopwatch says w seconds. A formula for $G(w)$ is

$$G(w) = 41 + 38\cos(2\pi w).$$

Maddy is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Maddy's head? (Assume Maddy is standing straight while watching the stopwatch.) *Remember to show your work.*

Solution: We are looking for when $41 + 38(\cos(2\pi w) > 60$.
First we find one time when $41 + 38(\cos(2\pi w) = 60$.

$$\begin{aligned} 38\cos(2\pi w) &= 19 \\ \cos(2\pi w) &= 1/2 \\ 2\pi w &= \arccos(1/2) \text{ (is one solution)} \\ 2\pi w &= \pi/3 \\ w &= 1/6 \end{aligned}$$

There are many ways to find the answer from here. One way is to note that since we are looking at a cosine function that has not been shifted horizontally, for $0 < w < 1/6$, the rope is above her head, so it's there for the first sixth of a second. By symmetry around the peak, the rope reached that height $1/6$ second before the timer started. Thus the rope is above her head for $2 \cdot 1/6 = 1/3$ second during each revolution.

Answer: $\frac{1}{3}$ **second**