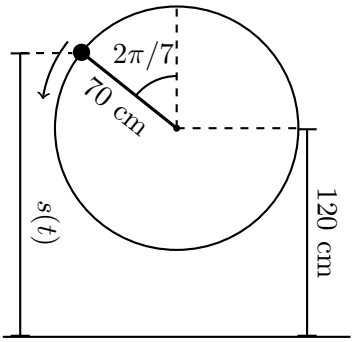
2. [11 points] Note that the situations in parts **a.** and **b.** are not related.

a. [6 points] In her latest trick, Dorraine swings a glow toy in a vertical circle (i.e., perpendicular to the ground). The glow toy starts to glow when it swings $2\pi/7$ radians past the top of the circle. The glow toy is attached to one end of a 70 cm rope, and Dorraine holds the other end at a constant height of 120 cm above the ground. The glow toy rotates at a constant rate, making 13 revolutions in 5 seconds. Let $s(t)$ be the height in cm above the ground of the glow toy t seconds after the glow toy starts to glow.



Find a formula for $s(t)$.

Note that there are many possible solutions. Two are given below.

Answer: $s(t) =$ $120 + 70\cos\left(\frac{26\pi}{5}t + \frac{2\pi}{7}\right)$ or $120 + 70\sin\left(\frac{26\pi}{5}t + \frac{2\pi}{7} + \frac{\pi}{2}\right)$

b. [5 points] Later, Dorraine swings a handmade toy. The height in cm above the ground of the handmade toy t seconds after she begins swinging it is given by

$$h(t) = 130 + 50\cos\left(\frac{10\pi}{7}t + \frac{\pi}{5}\right).$$

Compute the two smallest positive values of t at which the handmade toy was 160 cm above the ground. Clearly show each step of your work. Give your answers in exact form.

Solution: One solution is found using arccos:

$$130 + 50\cos\left(\frac{10\pi}{7}t + \frac{\pi}{5}\right) = 160$$

$$\cos\left(\frac{10\pi}{7}t + \frac{\pi}{5}\right) = 0.6$$

One solution is given by $\frac{10\pi}{7}t + \frac{\pi}{5} = \arccos(0.6)$

$$t = \frac{7}{10\pi}\left(\arccos(0.6) - \frac{\pi}{5}\right)$$

Verify that this is in fact the smallest positive solution.
The next positive solution of t (found using the unit circle or symmetry) is

$$t = \frac{7}{10\pi}\left(2\pi - \arccos(0.6) - \frac{\pi}{5}\right).$$

Answer: $t =$ $\frac{7}{10\pi}\left(\arccos(0.6) - \frac{\pi}{5}\right)$ and $\frac{7}{10\pi}\left(2\pi - \arccos(0.6) - \frac{\pi}{5}\right)$