4. [10 points] Consider the function f defined by $f(x) = \dfrac{(x+1.8)(x+2.1)}{(2x+1.8)(3x-6.9)(x+2.1)}$.

You do not have to show your work/reasoning on this problem. However, any work that you do show may be considered for partial credit.

a. [3 points] What is the domain of f?

Answer: all real numbers except -0.9, 2.3, and -2.1

b. [2 points] Find the equations of all vertical asymptotes of the graph of $y = f(x)$.
If there are none, write NONE.

Answer: $x = -0.9$ **and** $x = 2.3$

c. [2 points] Let $g(x) = e^{-0.4x}$.

Find the equations of all horizontal asymptotes of the graph of $y = \dfrac{g(x)}{f(x)}$.
If there are none, write NONE.

Solution: $g(x)$ is a positive exponential decay function and dominates any rational function as $x \to \infty$. In particular, $\lim\limits_{x\to\infty} \dfrac{g(x)}{f(x)} = 0$ and $\lim\limits_{x\to-\infty} \dfrac{g(x)}{f(x)} = \infty$ (DNE), so the only horizontal asymptote of the graph of $y = \dfrac{g(x)}{f(x)}$ is $y = 0$.

Answer: $y = 0$

d. [3 points] Find a formula for a rational function $h(x)$ such that $\lim\limits_{x\to\infty} \dfrac{f(x)}{h(x)} = 8$.

Solution: There are many possible answers. Some examples include:

- $h(x) = \dfrac{1}{8 \cdot 6 \cdot x} = \dfrac{1}{48x}$, and
- $h(x) = \dfrac{1}{8} f(x) = \dfrac{(x+1.8)(x+2.1)}{8(2x+1.8)(3x-6.9)(x+2.1)}$.

Answer: $h(x) = \dfrac{1}{48x}$