7. [10 points] Let $N(u) = \begin{cases} e + 3^{u^2+k} & \text{if } u < 1 \\ 5e\ln(e+u-1) & \text{if } u \geq 1, \end{cases}$ where k is a constant.

a. [6 points] Use the limit definition of the derivative to write an explicit expression for $N'(-2)$. *Your answer should not involve the letter N. Do not attempt to evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Answer: $N'(-2) = \lim_{h\to 0} \dfrac{e + 3^{(-2+h)^2+k} - (e + 3^{(-2)^2+k})}{h}$

b. [4 points] Find all values of k so that $N(u)$ is continuous at $u = 1$.
Show your work carefully, and leave your answer(s) in exact form.

Solution: To be continuous at $u = 1$, the left and right limits of $N(u)$ at 1 must both be equal to $N(1)$. In particular, k must satisfy the following equation:

$$e + 3^{1^2+k} = 5e\ln(e+1-1).$$

Solving for k, we find:

$$\begin{aligned} e + 3^{1+k} &= 5e \cdot 1 \\ 3^{1+k} &= 4e \\ \ln(3^{1+k}) &= \ln(4e) \\ (k+1)\cdot\ln(3) &= \ln(4) + \ln(e) \\ k + 1 &= \frac{\ln(4)+1}{\ln(3)} \\ k &= \frac{\ln(4)+1}{\ln(3)} - 1 \end{aligned}$$

Answer: $k = \dfrac{\ln(4)+1}{\ln(3)} - 1$

8. [7 points] Suppose w and q are continuous and invertible functions.
The table below shows many values of w and q^{-1} (the inverse of q).

s	-4.7	-3.3	-1.8	0.7	1.1	1.6	2.1	2.5	4.1	5.2
$w(s)$	4.1	2.5	1.4	0	-0.5	-1.8	-2	-3.1	-3.9	-4.7
$q^{-1}(s)$	-3.7	0.1	0.7	2.5	4.1	5.1	5.2	7.3	9.5	11.3

a. [2 points] Find $q^{-1}(w(-4.7))$.

Solution:
$q^{-1}(w(-4.7)) = q^{-1}(4.1) = 9.5$

Answer: 9.5

b. [2 points] Find $w(q(0.7))$.

Solution:
$w(q(0.7)) = w(-1.8) = 1.4$

Answer: 1.4

c. [3 points] Find the average rate of change of $q(x)$ between $x = 0.7$ and $x = 5.2$.
Be sure to show your work.

Solution: Average rate of change $= \dfrac{q(5.2) - q(0.7)}{5.2 - 0.7} = \dfrac{2.1 - (-1.8)}{4.5} = \dfrac{3.9}{4.5}$.

Answer: $\dfrac{3.9}{4.5} \approx 0.8667$