9. [12 points] Imtiyaz, Jacinta, and Katica, three food truck owners in San Francisco, gather to discuss the recent performances of their businesses. You may assume that all months have the same length. *Throughout this problem, be sure to show your work/reasoning carefully.*

a. [6 points] Jacinta feels that business has been slowing since she opened 6 months ago. She notes that she earned a total profit of $2470 during her 3rd month of business and a total profit of $1729 during her 5th month of business.

(i) Based on the data for months 3 and 5, if the profit of Jacinta's food truck were modeled by a linear function, what would the model predict her profit during her 9th month of business to be?

Solution: Linear functions have constant differences in output values for equally spaced input values. Profit changes by -741 (decreases by 741) dollars over two months so it changes by -1482 (decreases by 1482) dollars over four months.

Answer: _____ $247 _____

(ii) Based on this data, if the profit of Jacinta's food truck were instead modeled by an exponential function, what would the model predict her profit during her 9th month of business to be?

Solution: Exponential functions have constant ratios in output values for equally spaced input values. Ratio of profit two months apart is 0.7, so it the is 0.7^2 for profit four months apart.

Answer: _____ $847.21 _____

b. [3 points] Imtiyaz says that his profit for his 8th month of business was 45% higher than his profit in his 3rd month of business. If Imtiyaz's monthly profit increases by the same percentage every month, by what percent does it increase each month?

Solution: If a is the monthly growth factor, we know $a^5 = 1.45$, so $a = 1.45^{0.2}$. So the monthly percent growth is $1.45^{0.2} - 1 \approx 0.0771 = 7.71\%$.

Answer: _____ **about** 7.71 _____ percent

c. [3 points] Katica is really excited because her profit during her 12th month of business was 50% higher than in her 2nd month of business. If her profit is growing exponentially (as she hopes it is), in what month will Katica's profit be three times what it was in the 8th month? Round your answer to the nearest month.

Solution: The monthly growth factor $a = 1.5^{0.1}$ (from $a^{10} = 1.5$).
Solving for $a^{t-8} = 3$ (from $a^t = 3 \cdot a^8$) gives $t - 8 = \dfrac{\ln(3)}{0.1\ln(1.5)}$.
So $t = 8 + 10\dfrac{\ln(3)}{\ln(1.5)} \approx 8 + 27.1 = 35.1$.

Answer: _____ **during the 35th month** _____