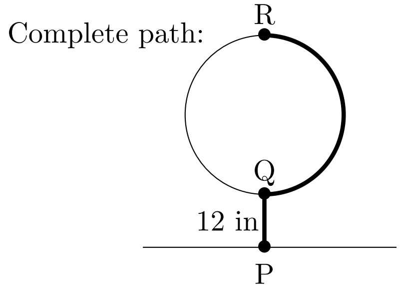
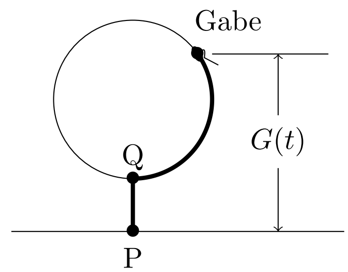
**2**. [13 points] After Blizzard left Arizona, Gabe the mouse found a large globe (a sphere) to climb. The globe has a diameter of 40 inches and it is attached to a 12-inch-long pole. Gabe starts at the base of the pole at point $P$. He climbs up to the bottom of the globe at point $Q$. He then climbs the globe along a semicircle until he stops at the top of the globe at point $R$ (see the diagram below). *Note that the diagram is not drawn to scale.*




**a**. [8 points] Assume that Gabe walks through the path at a velocity of 3 inches per second. Let $G(t)$ be Gabe's height above the ground (in inches) $t$ seconds after he started his climb at point $P$. Find a piecewise-defined formula for $G(t)$. *Be sure to include the domain for each piece.*

*Solution:* From point P to Q: It takes the ant 4 seconds to climb 12 inches at a velocity of 3 inches per second. During that time, the ant climbs at a constant rate of 3 inches per seconds starting at the floor, hence $G(t) = 3t$ for $0 \leq t \leq 4$.

From point Q to R: The distance $L$ along the semicircle traveled by the ant is $L = \frac{1}{2}(2\pi R)$, where $R$ is the radius of the circle. In this case $R = 20$ inches, then $L = 20\pi$. Hence it takes the ant $T = \frac{L}{3} = \frac{20\pi}{3}$ seconds to go from point Q to R a t a velocity of 3 inches per second. Its height is given by a sinusoidal function with midline at $k = 12 + 20 = 32$, amplitude $A = \frac{1}{2}(40) = 20$, period $P = 2T = \frac{40\pi}{3}$ and a minimum at $(4, 12)$. Hence $G(t) = 32 - 20\cos(B(t-4))$. The constant $B = \frac{2\pi}{P} = \frac{2\pi}{\frac{40\pi}{3}} = \frac{3}{20}$ for $4 \leq t \leq 4 + T$. Hence

$$G(t) = \begin{cases} 3t & \text{for} \quad 0 \leq t \leq 4. \\ 32 - 20\cos\left(\frac{3}{20}(t-4)\right) & \text{for} \quad 4 \leq t \leq 4 + \frac{20\pi}{3} \end{cases}$$

**b.** [5 points] After climbing the globe, Gabe jumps onto a small ferris wheel. Let $H(t)$ be his height, in inches, above the ground $t$ seconds after Gabe jumped, where

$$H(t) = 12 + 9\cos\left(\frac{\pi}{75}(t-120)\right).$$

Find the the *smallest* positive value of $t$ at which Gabe's height above the ground is 10.5 inches. Clearly show each step of your algebraic work. Give your answer in *exact* form.

*Solution:*

$$12 + 9\cos\left(\frac{\pi}{75}(t-120)\right) = 10.5$$

$$\cos\left(\frac{\pi}{75}(t-120)\right) = -\frac{1}{6}$$

$$\frac{\pi}{75}(t-120) = \cos^{-1}\left(-\frac{1}{6}\right) \qquad t_0 = 120 + \frac{75}{\pi}\cos^{-1}\left(-\frac{1}{6}\right)$$

$$\text{(smallest positive)} \qquad t_{ans} = t_0 - P = \frac{75}{\pi}\cos^{-1}\left(-\frac{1}{6}\right) - 30.$$

where the period of $H(t)$ is $P = \frac{2\pi}{\frac{\pi}{75}} = 150$.