5. [9 points] A company is hired to clean the trash accumulated in a lake.

1. Let $T(d)$ be the total amount of recyclable trash collected (in thousands of pounds) after they have cleaned for d days.
2. Let $R(s)$ be the revenue (in thousands of dollars) the company obtains from recycling s thousand pounds of recyclable trash.

Assume that the functions T and R are invertible and differentiable.

a. [4 points] Find a mathematical expression involving the functions T, R, T^{-1} and/or R^{-1} that represents each of the following sentences.

i) The revenue (in thousands of dollars) the company obtains from recycling all the trash collected from this lake if it takes the company 14 days to complete the job.

Solution: $R(T(14))$.

ii) The quantity, in thousands of pounds, of trash collected during the fifth day.

Solution: $T(5) - T(4)$

b. [2 points] Let $H(w)$ be the amount of recyclable trash collected (in *pounds*) during the first w *weeks* after the company started cleaning. Find a formula for $H(w)$ in terms of the functions T and R.

Solution: $H(w) = 1000T(7w)$.

c. [3 points] Circle the <u>one</u> statement below that is best supported by the equation

$$(R^{-1})'(20) = 3.5.$$

i) After the company has recycled enough trash to earn 20,000 dollars in revenue, if they recycle another thousand pounds of trash, then their revenue will be increased by about 3,500 dollars.

ii) Once the company recycles 20,000 pounds, the next thousand pounds of trash recycled will increase their revenue by about 3,500 dollars.

iii) The company earns 20,000 dollars for every 3,500 pounds of trash recycled.

iv) If the company recycles trash until they make 20,000 dollars in revenue, they need to recycle about 3,500 more pounds of trash to make an additional thousand dollars in revenue.

v) After the company has collected 20,000 pounds of recyclable trash, the amount of additional recyclable trash the company would have to collect to increase their revenue by 100 dollars is approximately 350 pounds.

Solution: IV)