7. [13 points] After testing different ingredients in their parents' garages, Imran and Nicole have recently opened new organic peanut butter companies.

a. [3 points] Two months after opening, Imran's company, Chunky Munky, has produced a total of 256 pounds of peanut butter. Imran thinks Chunky Munky produces peanut butter at a constant rate of 690 pounds every 6 months. Assuming Imran is correct, write a formula for $P(m)$, the total amount of peanut butter, in pounds, that Chunky Munky will have produced m months after opening.

Solution: Since the production increases at a constant rate, then $P(m)$ must be a linear function. The slope of $P(m)$ is $\frac{690}{6} = 115$ pounds per month. Since $P(2) = 256$, then using the point slope formula for linear functions we get

$$P(m) = 256 + 115(m - 2).$$

b. [4 points] Nicole's company, Lots O' Crunch, has produced a total of 182 pounds of peanut butter two months after opening and a total 454 pounds of peanut butter five months after opening. Nicole thinks that Lots O' Crunch produces peanut butter exponentially. Assuming Nicole is correct, write a formula for $Q(x)$, the total amount of peanut butter, in pounds, Lots O' Crunch will have produced x months after opening. *Decimal approximations must be rounded to at least three decimal places.*

Solution: We know that $Q(2) = 182$ and $Q(5) = 454$ where $Q(x) = ab^x$. Then

$$ab^5 = 454$$
$$ab^2 = 182$$
$$b^3 = \frac{454}{182}$$
$$b = \left(\frac{454}{182}\right)^{\frac{1}{3}} \approx 1.356 \quad \text{and} \quad a = \frac{182}{b^2} = \frac{182}{\left(\frac{454}{182}\right)^{\frac{2}{3}}} \approx 98.95.$$

Then $Q(x) = \frac{182}{\left(\frac{454}{182}\right)^{\frac{2}{3}}} \left(\left(\frac{454}{182}\right)^{\frac{1}{3}}\right)^x \approx 98.95(1.356)^x$.

Ann Arbor's leading local peanut butter company is Sticky PB Company. The total amount of peanut butter produced by Sticky PB Company m months after Chunky Munky opens is given by

$$S(m) = 1500e^{0.32m}.$$

c. [2 points] By what percent is Sticky PB Company's production growing every month? Round your answer to two decimal places.

Solution: Since $b = e^{.32}$, then $r = b - 1 = e^{.32} - 1 \approx 0.38$. Hence it grows by 38% every month.

d. [4 points] After a lot of analysis, Imran determines that Chunky Munky's total peanut butter production m months after opening is best modeled by the exponential function

$$C(m) = 100(1.6)^m.$$

According to this model, when will Chunky Munky and Sticky PB Company have produced the same amount of peanut butter? *Show all your work and leave your answer in exact form.*

Solution:
Method 1:

$$\begin{aligned}
1500e^{0.32m} &= 100(1.6)^m \\
\ln\left(1500e^{0.32m}\right) &= \ln\left(100(1.6)^m\right) \\
\ln(1500) + \ln(e^{.32m}) &= \ln(100) + \ln((1.6)^m) \\
\ln(1500) + 0.32m &= \ln(100) + m\ln(1.6) \\
0.32m - m\ln(1.6) &= \ln(100) - \ln(1500) \\
m(0.32 - \ln(1.6)) &= \ln(100) - \ln(1500) \\
m &= \frac{\ln(100) - \ln(1500)}{0.32 - \ln(1.6)}
\end{aligned}$$

Method 2:

$$1500e^{0.32m} = 100(1.6)^m$$

$$\frac{e^{.32m}}{(1.6)^m} = \frac{1}{15}$$

$$\left(\frac{e^{.32}}{1.6}\right)^m = \frac{1}{15}$$

$$\ln\left(\left(\frac{e^{.32}}{1.6}\right)^m\right) = \ln\left(\frac{1}{15}\right)$$

$$m\ln\left(\frac{e^{.32}}{1.6}\right) = \ln\left(\frac{1}{15}\right) \qquad \text{then} \qquad m = \frac{\ln\left(\frac{1}{15}\right)}{\ln\left(\frac{e^{.32}}{1.6}\right)}$$