3. [6 points]

a. [4 points] For which value(s) of the constant A is the function

$$R(t) = \begin{cases} 5(13)^{At} & \text{for } t < 2. \\ 20 - 3t^2 & \text{for } t \geq 2. \end{cases}$$

continuous? Find your answer algebraically and give your answer in exact form. If no such value exists, write "DNE". Show all your work step by step.

Solution: The function $R(t)$ is continuous on $(-\infty, 2)$ and $(2, \infty)$. In order for $R(t)$ to be continuous at $t = 2$, $R(t)$ has to satisfy $R(2) = \lim_{t\to 2} R(t)$. Since $R(2) = 8 = \lim_{t\to 2^+} R(t)$, then it is only necessary that $\lim_{t\to 2^-} R(t) = \lim_{t\to 2^-} 5(13)^{At} = 5(13)^{2A} = 8$. This yields

$$\begin{aligned} 5(13)^{2A} &= 8 \\ 13^{2A} &= \frac{8}{5} = 1.6 \\ 2A\ln(13) &= \ln(1.6) \\ A &= \frac{\ln(1.6)}{2\ln(13)} \end{aligned}$$

Answer: $A = \dfrac{\ln(1.6)}{2\ln(13)}$

b. [2 points] A different function, $f(d)$, has the property that $\lim_{d\to\infty} f(d) = 10$. What is the value of $\lim_{d\to\infty} 4f(2d - 14) + 9$?

Write "DNE" if the limit does not exist or "NI" if there is not enough information to answer the question. You do not need to show your work.

Solution: Since $2d - 14$ tends to infinity as d tends to infinity, then $\lim_{d\to\infty} f(2d - 14) = 10$. Hence $\lim_{d\to\infty} 4f(2d - 14) + 9 = 4(10) + 9 = 49$.

Answer: 49