5. [16 points] On a particularly cold winter day, Nia decides to turn on her gas powered heater at 5:00 pm. Over the next few hours, she records the temperature of her house and the amount of gas that the heater has used. She also notices that the temperature of her house seems to affect how loudly her dog barks. Nia uses the following three functions to model her observations:

- $T(t)$ represents the temperature (in degrees Fahrenheit) of Nia's house t minutes after 5:00 pm.
- $G(t)$ represents the amount of gas (in cubic feet) that the heater used during the first t minutes after 5:00 pm.
- $B(x)$ represents the sound intensity (in decibels) of the dog's barks when the temperature of her house is x degrees Fahrenheit.

You may assume that T, G and B are invertible and differentiable functions.

a. [10 points] Find a mathematical expression for each of the quantities below using the functions T,G,B, their inverses and/or their derivatives.

(i) The number of minutes the heater had been on when it had used 2 cubic feet of gas.

Solution:

Answer: $G^{-1}(2)$

(ii) The sound intensity (in decibels) of the dog's barks at 6:20 pm.

Solution:

Answer: $B(T(80))$

(iii) The approximate change in temperature of Nia's house (in degrees Fahrenheit) between 5:20 pm and 5:21pm.

Solution:

Answer: $T'(20)$

(iv) The amount of gas (in cubic feet) used by the heater between 5:30 pm and 7:00 pm.

Solution:

Answer: $G(120) - G(30)$

(v) The temperature (in degrees Fahrenheit) of Nia's house k minutes before 7:00 pm.

Solution:

Answer: $T(120 - k)$

b. [3 points] Complete the sentence below with a valid interpretation of the equation $T'(60) = 0.88$.

Solution:
In the first 15 seconds after 6:00 pm, the temperature of Nia's house **increases by about** $0.22°$ **F.**

This problem continues on the next page.

The statement of the problem has been included for your convenience.

On a particularly cold winter day, Nia decides to turn on her gas powered heater at 5:00 pm. Over the next few hours, she records the temperature of her house and the amount of gas that the heater has used.

Let $G(t)$ represent the amount of gas (in cubic feet) that the heater used during the first t minutes after 5:00 pm.

c. [3 points] Circle the <u>one</u> sentence that gives a valid interpretation of the equation

$$(G^{-1})'(3) = 72.$$

Solution:

    (A) Nia's heater has used 3 cubic feet of gas at 6:12 pm.

    (B) The amount of gas used by the heater between 5:03 pm and 5:04 pm will be approximately 72 cubic feet.

    (C) Once the heater has used 72 cubic feet of gas, it takes about 3.6 minutes for it to use an additional 0.05 cubic feet of gas.

    (D) It will take approximately 3.6 minutes for the amount of gas used by the heater to increase

    (E) The heater uses 3 cubic feet of gas every 72 minutes.

6. [5 points] Let

$$A(w) = 5\sin(3w) - 4^{-w}.$$

Use the limit definition of the derivative to write an explicit expression for $A'(2)$. *Your answer should not involve the letter A. Do not attempt to evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Solution: $A'(2) = \lim\limits_{h \to 0} \dfrac{5\sin(3(2+h)) - 4^{-(2+h)} - \left(5\sin(6) - 4^{-2}\right)}{h}$