7. [9 points] During the production of electricity from fossil fuel, nitrogen oxides are produced. The 1990 amendments to the Clean Air Act established the Acid Rain Program to reduce power plant nitrogen oxides and other emissions. A city in Michigan estimated that the annual nitrogen oxide emissions from its power plants were 191.4 thousand tons in 1990. Let $N(t)$ be a function that models the estimated annual nitrogen oxide emissions from that city's power plants (in thousand of tons) t years after 1970.
Find a formula for $N(t)$ assuming:

- The function $N(t)$ is continuous on its domain $[0, 47]$.
- The amount of annual nitrogen oxide emissions increased at a constant rate of 8 thousand tons every five years in between 1970 and 1990.
- The amount of annual nitrogen oxide emissions decayed exponentially by 20 percent every 3 years after 1990.

Solution:

- On $[0, 20]$: $N(t)$ is linear with rate of change $\frac{8}{5}$ thousands of tons of nitrogen oxide per year with $N(20) = 191.4$. Hence $N(t) = \frac{8}{5}(t - 20) + 191.4$.
- On $[20, 47]$: $N(t)$ is exponential. If $N(t) = ab^t$, then we know that $ab^{20} = 191.4$ and $ab^{23} = 191.4(0.8)$. Dividing both equations

$$\frac{ab^{23}}{ab^{20}} = b^3 \qquad \text{and} \qquad \frac{ab^{23}}{ab^{20}} = \frac{1.91.4(0.8)}{191.4} = 0.8.$$

Then $b^3 = 0.8$ yields $b = \sqrt[3]{0.8}$. Plugging into $ab^{20} = 191.4$ yields $a = 191.4(0.8)^{-\frac{20}{3}}$

$$N(t) = \begin{cases} \frac{8}{5}(t - 20) + 191.4 & \text{for} \quad 0 \le t < 20 \\ 191.4(0.8)^{\frac{1}{3}(t-20)} & \text{for} \quad 20 \le t \le 47 \end{cases}$$