9. [11 points] A group of marine biologists are studying life in the Challenger Deep, the deepest known point in the world's ocean. They use a special submarine to take samples of sea water for their study. Let $S(t)$ be the depth of the submarine (in miles) t minutes after it started collecting sea water samples. In this problem, depth will always be a positive number.

a. [5 points] Find a formula for $S(t)$ assuming that:

- $S(t)$ is a sinusoidal function.
- The submarine rises in 4 hours from a maximum depth of 6 miles to half a mile below the sea level (the closest point it gets to the surface).
- The submarine reaches its maximum depth 30 minutes after it starts taking sea water samples.

Solution: We know that $S(t)$ has a maximum at $t = 30$, then $S(t) = A\cos(B(t-30)) + k$ with $A > 0$. The amplitude of $S(t)$ is $\frac{6-0.5}{2} = 2.75$. The period of $S(t)$ is 480 (8 hours) and its midline is $y = \frac{6+0.5}{2} = 3.25$. Then $A = 2.75$, $B = \frac{2\pi}{480} = \frac{\pi}{240}$ and $k = 3.25$.

Answer: $S(t) = 2.75\cos\left(\frac{\pi}{240}(t-30)\right) + 3.25$

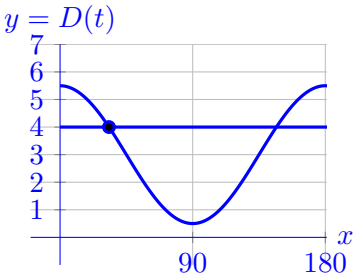
b. [6 points] During a second expedition, the depth of the submarine (in miles) is given by the function

$$D(t) = 3 + 2.5\cos\left(\frac{\pi}{90}t\right)$$

where t represents the time in minutes after the submarine started collecting samples. Once the submarine reaches a depth of 4 miles for the first time, how much time passes before it is at a depth of 4 miles for the second time? Your answer must be in exact form. Show all your work and include units.

Solution: Setting $D(t) = 4$ and solving for t:

$$3 + 2.5\cos\left(\frac{\pi}{90}t\right) = 4$$
$$\cos\left(\frac{\pi}{90}t\right) = 0.4$$
$$\frac{\pi}{90}t = \cos^{-1}(0.4) \quad \text{then} \quad t_1 = \frac{90}{\pi}\cos^{-1}(0.4).$$



Answer: $180 - 2t_1 = 180 - \frac{180}{\pi}\cos^{-1}(0.4)$

A different approach using two consecutive solutions t_1 and t_2 where find t_2 from the equation

$$\frac{\pi}{90}t = 2\pi - \cos^{-1}(0.4) \quad \text{then} \quad t_2 = \frac{90}{\pi}(2\pi - \cos^{-1}(0.4)) = 180 - \frac{90}{\pi}\cos^{-1}(0.4)$$

yields **Answer:** $t_2 - t_1 = 180 - \frac{180}{\pi}\cos^{-1}(0.4)$