1. [8 points] The Amazing Wanda is performing a magic act.

a. Let $V(t)$ be the volume, in decibels (dB), of the audience's applause t seconds after the beginning of the act.

i. [2 points] At time $t = 0$, the audience is already clapping at a volume of 52 dB. During Wanda's first trick, which lasts 45 seconds, the volume of the audience's applause increases at a constant rate of 0.4 dB per second. Write a formula for the function $V(t)$ during the first trick.

Answer: $V(t) =$ $0.4t + 52$ for $0 \le t \le 45$

ii. [4 points] During Wanda's second trick, which begins at $t = 45$ and lasts until the end of the act at time $t = 95$, the volume of the audience's applause increases by 1.2% every second. Write a piecewise formula for the function $V(t)$ on the interval $[0, 95]$. Make sure that $V(t)$ is a continuous function.

Solution: The first piece of the formula for $V(t)$ was the answer to part i.

The function $V(t)$ is exponential for $45 \le t \le 95$ with growth factor 1.012. Note that at time $t = 45$, Wanda's audience is clapping at a volume of $0.4(45) + 52 = 70$ dB.

Approach 1: Since the constant percent growth begins at time $t = 45$, we need to shift the exponential function with initial value 70 forward (to the right) by 45 seconds. This gives

$$70(1.012)^{t-45} = 70(1.012)^{-45}(1.012)^t = \frac{70}{1.012^{45}}(1.012)^t.$$

Note that this answer (which is exact) can be approximated by either of the following

$$70(1.012)^{t-45} \approx 40.924(1.012)^t \text{ or } 70(1.012)^{t-45} \approx 40.924e^{0.0119t}$$

Approach 2: We can also find a formula for the exponential piece of $V(t)$ by using the facts that $V(45) = 70$ and $V(t)$ is continuous. Solving for a in the resulting equation $70 = a(1.012)^{45}$. gives the value $a = \frac{70}{1.012^{45}}$ resulting in the same formula as the previous approach.

Answer: $V(t) = \begin{cases} 0.4t + 52 & \text{for } 0 \le t \le 45 \\ 70(1.012)^{t-45} & \text{for } 45 < t \le 95 \end{cases}$

b. [2 points] A few minutes after her act, Wanda returns to the stage for an encore performance. Let $W(s)$ be the volume, in dB, of the audience's applause s seconds after the encore begins. A table of some values of $W(s)$ is given below.

s	0	2	3
$W(s)$	3.00	3.60	4.32

Could $W(s)$ be an exponential function? Circle your answer below. Show your work to justify your answer.

Solution: No, $W(s)$ could not be exponential. Over the interval $[2, 3]$, the function grows by a factor of $4.32/3.60 = 1.2$. If $W(s)$ were exponential, it would therefore have to grow by a factor of $(1.2)^2$ over the interval $[0, 2]$. But $3.6/3 = 1.2 \neq (1.2)^2$.

Answer: YES **NO**