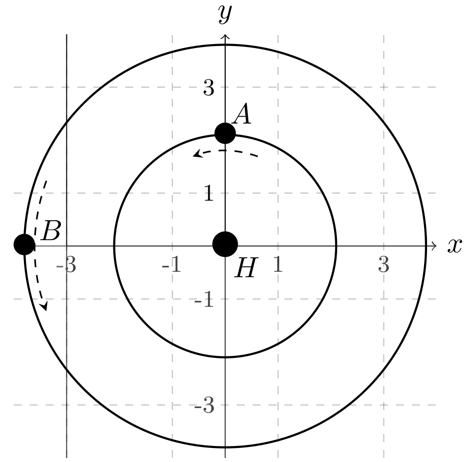
3. [10 points] Horatio the Daring is performing a dangerous stunt. Helicopter A and Helicopter B are circling around Horatio to film the event. Let t be the amount of time, in minutes, since the start of Horatio's stunt.

A top-down view of the flight paths is shown at right. The locations of the helicopters at $t = 0$ are labeled A and B, respectively, and Horatio's location is labeled H (at the origin).

All distances are measured in kilometers (km). The helicopters are flying counter-clockwise around Horatio in perfect circles at a constant height above the ground.



a. [4 points] Helicopter A moves at a constant speed of 0.7 km/min around a circle of radius 2.1 km. Write a formula for the function $a(t)$ that gives the y-coordinate of Helicopter A at time t.

Solution: A circle of radius 2.1 km has circumference 4.2π km. If the helicopter flies at a constant speed of 0.7 km/min, then it takes the helicopter $(4.2\pi)/0.7 = 6\pi$ minutes to fly all the way around the circle. The period of $a(t)$ is therefore 6π minutes.

Note that the y-coordinate of Helicopter A begins at its maximum value, so we can model $a(t)$ using a cosine function without a horizontal shift.

Answer: $a(t) = 2.1\cos\left(\frac{t}{3}\right)$ **or** $2.1\sin\left(\frac{t}{3}+\frac{\pi}{2}\right) = 2.1\sin\left(\frac{1}{3}\left(t+\frac{3\pi}{2}\right)\right)$

b. [6 points] The x-coordinate of Helicopter B at time t is given by the formula

$$b(t) = -3.8\cos\left(\frac{\pi}{32}t\right).$$

Find **all** values of t during the first hour of the stunt at which the location of Helicopter B has x-coordinate less than or equal to -3. Give your answer as one or more intervals, with endpoints in exact form.

Solution: First we find the times in the first hour ($0 \le t \le 60$) when $b(t) = -3$. We know

$$-3.8\cos\left(\frac{\pi}{32}t\right) = -3$$

$$\cos\left(\frac{\pi}{32}t\right) = \frac{3}{3.8},$$

so one solution is given by $\frac{\pi}{32}t = \arccos\left(\frac{3}{3.8}\right)$

which gives $t = \frac{32}{\pi}\arccos\left(\frac{3}{3.8}\right) \approx 6.73$ min.

The period of $b(t)$ is $2\pi/(\pi/32) = 64$ min, and because $b(t)$ begins at its minimum value, we can use symmetry to see that the second (and final) time at which $b(t) = -3$ during the first hour is $t = 64 - \frac{32}{\pi}\arccos\left(\frac{3}{3.8}\right) \approx 57.27$ minutes.
The x-coordinate of Helicopter B is greater than -3 between these two times.

Answer: $\left[0, \frac{32}{\pi}\arccos\left(\frac{3}{3.8}\right)\right]$ **and** $\left[64-\frac{32}{\pi}\arccos\left(\frac{3}{3.8}\right), 60\right]$