5. [12 points] A weather balloon is launched and heads straight up away from the ground. Let $R(t)$ be the height, in kilometers, of the balloon above the ground t minutes after its launch. The function $R(t)$ is invertible and differentiable.

t	1	3	9	18	35	45	60	63	86
$R(t)$	0.01	0.19	0.4	0.84	2.3	3	3.7	4.1	8.9

a. [2 points] On which of the following intervals could $R(t)$ be concave up on the entire interval? Circle **all** correct answers.

$[1,9]$ $\boxed{[3,18]}$ $\boxed{[9,35]}$ NONE OF THESE

b. [2 points] Find the balloon's average velocity between times $t = 3$ and $t = 18$. Show work and *include units.*

Solution: The balloon's average velocity over this time period is given by

$$\frac{R(18) - R(3)}{18 - 3} = \frac{0.84 - 0.19}{18 - 3} = \frac{0.65}{15} = \frac{13}{300}.$$

Answer: $\frac{0.84 - 0.19}{18 - 3} = \frac{13}{300} \approx 0.0433$ **km/min**

c. [3 points] Estimate the balloon's instantaneous velocity at $t = 63$. Show work and *include units.*

Solution: The balloon's instantaneous velocity at $t = 63$ is $R'(63)$.
The closest given time to $t = 63$ is $t = 60$, so we use the average rate of change of R over $[60, 63]$ to estimate $R'(63)$. (Note that $t = 86$ is very far from $t = 63$ when compared to $t = 60$.)

$$R'(63) \approx \frac{R(63) - R(60)}{63 - 60} = \frac{4.1 - 3.7}{63 - 60} = \frac{0.4}{3} = \frac{4}{30} \approx 0.133 \text{ km/min}$$

Answer: **approximately** 0.133 **km/min**

d. [3 points] Estimate $(R^{-1})'(3)$. Show work and *include units.*

Solution: The closest given distances to 3 km are 2.3 km and 3.7 km. We will estimate $(R^{-1})'(3)$ by taking the average rate of change of R^{-1} over the interval $[2.3, 3.7]$.

$$(R^{-1})'(3) \approx \frac{R^{-1}(3.7) - R^{-1}(2.3)}{3.7 - 2.3} = \frac{60 - 35}{3.7 - 2.3} = \frac{25}{1.4} = \frac{125}{7} \approx 17.86 \text{ min/km}$$

It would also be okay to use the interval $[2.3, 3]$ or $[3, 3.7]$, which would give about 14.3 and 21.4 min/km, respectively. The average of these two estimates would give the estimate we found above.

Answer: $(R^{-1})'(3) \approx$ 17.86 **min/km**

e. [2 points] Let $M(s)$ be the height, in meters, of the balloon above the ground s seconds after its launch. Find a formula for $M(s)$ in terms of R and s. (There are 1000 meters in one kilometer.)

Answer: $M(s) = 1000R\left(\frac{1}{60}s\right)$