

8. [12 points] The size of the harvest at a kale farm is a function of the total amount of compost the farm uses in the fields.
- Let  $K(c)$  be the size (as measured by weight) of the farm's kale harvest, in tons, when the farm uses  $c$  cubic meters ( $\text{m}^3$ ) of compost.
  - Let  $P(h)$  be the farm's profit, in thousands of dollars, when their kale harvest is  $h$  tons.

The functions  $K(c)$  and  $P(h)$  are differentiable, and the function  $P(h)$  is invertible.

- a. [2 points] Using a complete sentence, give a practical interpretation of the equation

$$P^{-1}(86) = 53.$$

*Solution: In order for the kale farm to make 86 thousand dollars in profit, they need to harvest 53 tons of kale.*

- b. [3 points] Write a single equation involving  $K$ ,  $P$ , and/or  $P^{-1}$  that represents the following statement.

*If the farm uses  $1125 \text{ m}^3$  of compost, their profit will be twice as large as if they had used  $700 \text{ m}^3$  of compost.*

**Answer:**  $P(K(1125)) = 2P(K(700))$

- c. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$K'(950) = 0.2.$$

*If the farm uses  $955 \text{ m}^3$  of compost instead of  $950 \text{ m}^3$ , ...*

*Solution: ... they would harvest roughly 1 additional ton of kale.*

- d. [4 points] Write a single equation involving the derivative function(s)  $K'$ ,  $P'$ , and/or  $(P^{-1})'$  that represents the following statement.

*In order for the farm's profit to be \$101,500 rather than \$100,000, their kale harvest must be about 0.9 tons larger.*

**Answer:**  $(P^{-1})'(100) = 0.6$