9. [10 points] Parts a. – c. below are not related. You do not need to show work on this page, but partial credit may be earned for work shown.

a. [4 points] A portion of the graph of a polynomial function \( q(x) \) is shown below. Find a possible formula for \( q(x) \) of the smallest possible degree. Assume that all of the key features of the graph are shown.

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Solution: We see that the degree of \( q(x) \) must be even. The zeros of \( q(x) \) are \(-1, 2, \) and \(4\). Note that 2 is a double zero. We use the point \((0, 4)\) to find the leading coefficient.

\[
q(x) = C(x + 1)(x - 2)^2(x - 4)
\]
\[
4 = C(0 + 1)(0 - 2)^2(0 - 4)
\]
\[-\frac{1}{4} = C\]

Answer: \( q(x) = \frac{-1}{4}(x + 1)(x - 2)^2(x - 4) \)
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b. [3 points] Find the formula for a rational function \( r(x) \) that has a hole with an \( x \)-value of 5, a vertical asymptote at \( x = 1 \), and a horizontal asymptote at \( y = -2 \).

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Solution: Note that the factor of \( x \) in the numerator could be replaced by any degree 1 factor \((x + B)\) with \( B \) a constant.

Answer: \( r(x) = \frac{-2(x - 5)x}{(x - 5)(x - 1)} \)
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c. [3 points] Consider the function

\[
z(x) = \frac{4^{-x} - 2x^2}{15x + 3x^2}.
\]

Find \( \lim_{x \to \infty} z(x) \) and \( \lim_{x \to -\infty} z(x) \). If the value does not represent a real number (including the case of limits that diverge to \( \infty \) or \( -\infty \)), write “DNE” or “does not exist.”

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Solution: Note that \( 4^{-x} = \left(\frac{1}{4}\right)^x \), so \( \lim_{x \to \infty} 4^{-x} = 0 \) and \( \lim_{x \to -\infty} 4^{-x} \) does not exist (as \( 4^{-x} \) diverges to \( \infty \) as \( x \to -\infty \)).

Answer: \( \lim_{x \to \infty} z(x) = \frac{-2}{3} \) and \( \lim_{x \to -\infty} z(x) = \text{DNE} \)
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