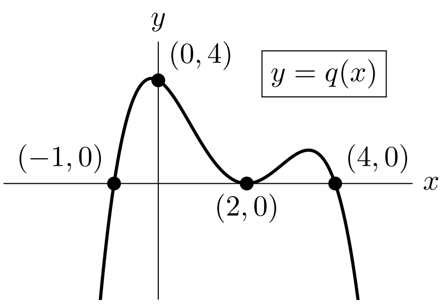
9. [10 points] Parts **a.** – **c.** below are not related. You do not need to show work on this page, but partial credit may be earned for work shown.

a. [4 points] A portion of the graph of a polynomial function $q(x)$ is shown below. Find a possible formula for $q(x)$ of the smallest possible degree. Assume that all of the key features of the graph are shown.



Solution: We see that the degree of $q(x)$ must be even. The zeros of $q(x)$ are -1, 2, and 4. Note that 2 is a double zero. We use the point $(0, 4)$ to find the leading coefficient.

$$q(x) = C(x+1)(x-2)^2(x-4)$$
$$4 = C(0+1)(0-2)^2(0-4)$$
$$-\frac{1}{4} = C$$

Answer: $q(x) =$ $-\frac{1}{4}(x+1)(x-2)^2(x-4)$

b. [3 points] Find the formula for a rational function $r(x)$ that has a hole with an x-value of 5, a vertical asymptote at $x = 1$, and a horizontal asymptote at $y = -2$.

Solution: Note that the factor of x in the numerator could be replaced by any degree 1 factor $(x + B)$ with B a constant.

Answer: $r(x) =$ $\frac{-2(x-5)x}{(x-5)(x-1)}$

c. [3 points] Consider the function

$$z(x) = \frac{4^{-x} - 2x^2}{15x + 3x^2}.$$

Find $\lim_{x\to\infty} z(x)$ and $\lim_{x\to-\infty} z(x)$. If the value does not represent a real number (including the case of limits that diverge to ∞ or $-\infty$), write "DNE" or "does not exist."

Solution: Note that $4^{-x} = \left(\frac{1}{4}\right)^x$, so $\lim_{x\to\infty} 4^{-x} = 0$ and $\lim_{x\to-\infty} 4^{-x}$ does not exist (as 4^{-x} diverges to ∞ as $x \to -\infty$).

Answer: $\lim_{x\to\infty} z(x) =$ $-\frac{2}{3}$ and $\lim_{x\to-\infty} z(x) =$ **DNE**