2. [12 points] Johannes takes a train from Buenos Aires to his countryside ranch, a distance of 1000 kilometers, traveling in a straight line. During the trip to the ranch, the train stops once, at the town of Rivadavia. After Johannes arrives at the ranch he realizes that he left an important book in Buenos Aires, so he returns to the city on an express train, which travels directly back to Buenos Aires on the same track with no stops.

Let $J(t)$ be Johannes's distance from Buenos Aires, in kilometers (km), at time t hours (h) after the train begins moving. Some values of $J(t)$ are shown in the table below.

t	0	3	7	8	9	12	14	16	18
$J(t)$	0	450	650	650	750	950	950	650	0

a. [2 points] How far is Rivadavia from Johannes's countryside ranch? Include units.

Solution: $1000 - 650 = 350$ km

(As stated in the problem, Johannes's ranch is 1000 km from Buenos Aires. The train only stops once on the way to the ranch, so the train must be stopped in Rivadavia between times $t = 7$ and $t = 8$. So Rivadavia is 650 km from Buenos Aires, and the distance between the ranch and Rivadavia is $1000 - 650 = 350$ km.)

b. [2 points] What is the average velocity of the train between $t = 3$ and $t = 7$? Include units.

Solution: $\frac{650-450}{7-3} = 50$ km/hr

c. [2 points] Estimate the instantaneous velocity of the train 9 hours into Johannes's trip. Include units.

Solution: There are several ways to reasonably estimate the instantaneous velocity at $t = 9$ using the table. One such estimate is given by the average velocity from $t = 8$ to $t = 9$.
9 hours into the trip, the instantaneous velocity is approximately $\frac{750-650}{9-8} = 100$ km/hr

d. [2 points] For which of the following time intervals is the instantaneous velocity negative at some point in the interval? Give your answer as a list of one or more intervals, or write NONE.

$[8, 12]$ $\boxed{[12, 14]}$ $\boxed{[8, 16]}$ $\boxed{[14, 18]}$

Solution: Some explanation (not necessary to earn full credit) follows.
Since the train turns around (reaches the ranch) between $t = 12$ and $t = 14$, the instantaneous velocity is never negative from $t = 0$ to $t = 12$ (including the interval $[8, 12]$) and is always negative from $t = 14$ to $t = 18$ (including the interval $[14, 18]$). It is also negative at some time in the interval $[12, 14]$, after the time when the train turns around.

e. [2 points] If the average velocity of the train on its return trip from the ranch to Buenos Aires was 200 km/h, and it arrived in Buenos Aires at exactly $t = 18$, at what time t did the train depart?

Solution: Let t be the time when the train left the ranch to return to Buenos Aires. On the return trip, the time elapsed is $18 - t$, the displacement is $1000 - 0$, and the average velocity is is 200 km/hr. So we have $\frac{1000-0}{18-t} = 200$ and find that the train left at time $t = 13$.

f. [2 points] Could the graph of $J(t)$ be concave up for the entire interval $0 \le t \le 7$? *Briefly explain your reasoning.*

Solution: The train's average velocity on the interval $0 \le t \le 3$ is $\frac{450-0}{3-0} = 150$ while its average velocity on the interval $3 \le t \le 7$ is $\frac{650-450}{7-3} = 50$. This means that the velocity cannot always be increasing, so the graph cannot be concave up for the entire interval. (Note that this conclusion agrees with our physical intuition: the train must slow down as it comes to a stop at the station in Rivadavia, which happens no later than time $t = 7$.)