4. [11 points]

t	1	4	10
$R(t)$	2	6	18

a. [6 points] Consider the given table of values for the function $R(t)$.

i. Could $R(t)$ be a linear function? Write YES or NO, show your work, and briefly explain your answer.

Solution: If $R(t)$ were linear then its average rate of change over any interval would be the same. However, the average rate of change on $[1, 4]$ is $\frac{6-2}{4-1} = \frac{4}{3}$ whereas the average rate of change on $[4, 10]$ is $\frac{18-6}{10-4} = \frac{12}{6} = 2$. Since these rates of change are not equal, $R(t)$ cannot be linear.

ii. Could $R(t)$ be an exponential function? Write YES or NO, show your work, and briefly explain your answer.

Solution: If $R(t)$ were exponential then its growth factor over any interval would be constant. On the one hand, the growth factor from 1 to 4 would be the solution to the equation $a^{4-1} = \frac{6}{2} = 3$, that is, $a = 3^{1/3}$. On the other hand, the growth factor from 4 to 10 would be the solution to the equation $a^{10-4} = \frac{18}{6} = 3$, that is, $a = 3^{1/6}$. Since these growth factors are not the same, $R(t)$ cannot be exponential.

b. [5 points] Consider a different function $S(t)$, which is equal to 5 at $t = 0$, and decreases by 40% every 4 units of time. For which value of t will $S(t)$ be equal to 1? *Show every step of your work, and give your final answer in exact form.*

Solution: The growth factor a is the solution to the equation $a^4 = 1 - 0.4$, that is, $a = (0.6)^{1/4}$. Since the initial value is $S(0) = 5$, a formula for $S(t)$ is $S(t) = 5\big((0.6)^{1/4}\big)^t = 5(0.6)^{t/4}$. Now we solve for t in the equation $S(t) = 1$.

$$\begin{aligned} 5(0.6)^{t/4} &= 1 \\ 0.6^{t/4} &= 0.2 \\ \ln(0.6^{t/4}) &= \ln(0.2) \\ \frac{t}{4} \cdot \ln(0.6) &= \ln(0.2) \\ t &= \frac{4\ln(0.2)}{\ln(0.6)} \end{aligned}$$