

6. [11 points] Define the following functions for an airplane taking off from a certain airport.
- Let  $H(t)$  be the height above sea level, in kilometers (km), of the airplane  $t$  minutes after takeoff.
  - Let  $T(k)$  be the temperature of the air outside the airplane, in degrees Celsius ( $^{\circ}\text{C}$ ), at a height of  $k$  kilometers above sea level.

The functions  $H(t)$  and  $T(k)$  are differentiable and invertible.

- a. [2 points] Use a complete sentence to give a practical interpretation of the equation  $H^{-1}(6) = 5$ .

*Solution:* The airplane takes 5 minutes to reach a height of 6 kilometers.

- b. [3 points] Write a single equation representing the following statement in terms of the functions  $H, T$ , and/or their inverses:

*The temperature of the air outside the airplane fell by  $12^{\circ}\text{C}$  in the first five minutes after takeoff.*

*Solution:*

$$T(H(5)) - T(H(0)) = -12$$

- c. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$T'(9) = -10.$$

*As the plane climbs from 8.8 km above sea level to 9 km above sea level...*

*Solution:* “the temperature drops by about  $2^{\circ}\text{C}$ .”

- d. [3 points] Which of the following gives a valid interpretation of the equation  $(H^{-1})'(4) = 0.5$ ? Write down the Roman numeral corresponding to your choice. There is only one correct answer.
- When the plane is at a height of 4 km, the temperature of the air outside the plane will decrease by about 0.5 degrees Celsius as the plane climbs an additional kilometer.
  - It will take approximately 30 seconds for the airplane to climb from a height of 4 km to a height of 5 km.
  - Four minutes into its flight, the plane will increase its height by about 0.5 km in the next minute.
  - Once the plane has reached a height of 4 km, it will take about one minute to climb an additional 0.5 km.

*Solution:* The correct answer is ii.