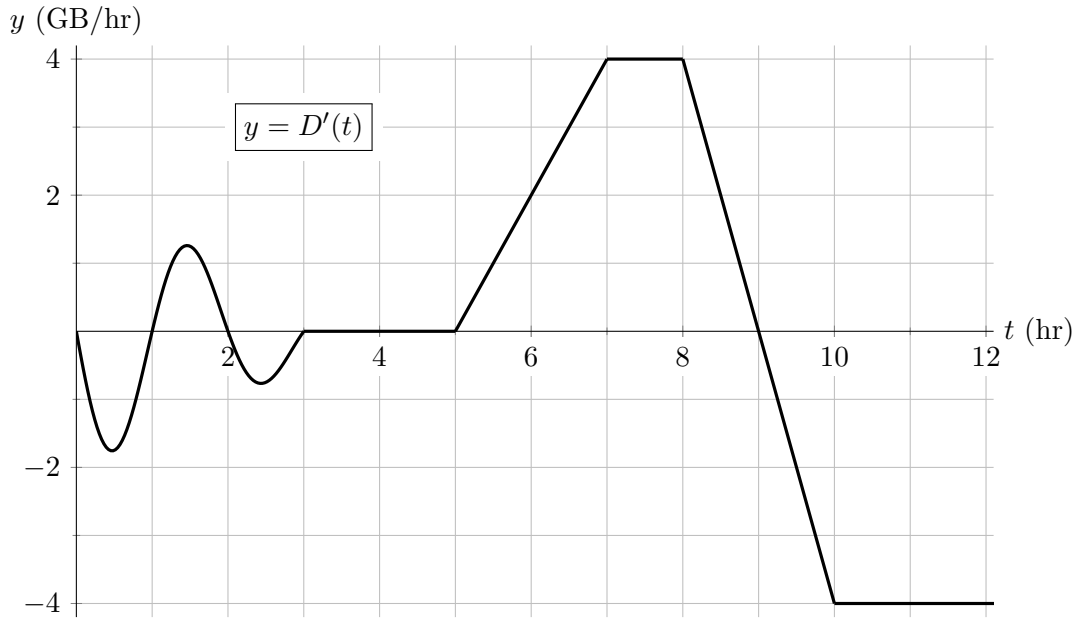
9. [8 points] The server for a website stores user data. Let $D(t)$ be the amount of user data stored on the server, in gigabytes (GB), at time t hours after noon. Below is a portion of the graph of $D'(t)$, the **derivative** of $D(t)$. The function $D'(t)$ is

- constant for $3 \le t \le 5$, for $7 \le t \le 8$, and for $t \ge 10$, and is
- linear for $5 \le t \le 7$ and for $8 \le t \le 10$.



a. [2 points] On which of the following intervals of t is the amount of user data stored on the server increasing for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(0.5, 1.5)$ $\boxed{(1, 2)}$ $\boxed{(7, 8)}$ $(10, 12)$

Solution: (The amount of user data is increasing when its derivative, the function $D'(t)$ graphed above, is positive.)

b. [2 points] When the amount of user data on the server is changing faster than 2 GB/hr, either increasing or decreasing, the server is said to be in an "excited state." How many hours, between noon and midnight, does the server spend in an excited state?

Solution: The server spends $\boxed{5 \text{ hours}}$ in an excited state.
(This question is asking for the times t when $D'(t) > +2$ or $D'(t) < -2$. The graph lies above the line $y = +2$ for $6 < t < 8.5$and lies below the line $y = -2$ for $6 < t < 8.5$.)

c. [2 points] The server hibernates when the amount of user data is not changing. How many hours, between noon and midnight, does the server spend in hibernation?

Solution: The amount of user data $D(t)$ is not changing exactly when the derivative $D'(t)$ equals zero. The total time in hibernation is therefore $5 - 3 = \boxed{2 \text{ hours}}$.

d. [2 points] At midnight, 450 GB of data is stored on the server. If the rate of change of user data stays the same from midnight to 5 am the following morning, how much user data will be stored on the server at 5 am?

Solution: The graph above shows that $D'(12) = -4$, so at midnight, the amount of data is changing at a rate of -4 GB/hr. From midnight to 5 am the total change in the amount of data is therefore $5 \text{ hr} \cdot (-4 \text{ GB/hr}) = -20$ GB. Since the server has 450 GB of data at midnight, there must be $450 - 20 = \boxed{430 \text{ GB}}$ of data at 5 am.