9. [8 points] The server for a website stores user data. Let \( D(t) \) be the amount of user data stored on the server, in gigabytes (GB), at time \( t \) hours after noon. Below is a portion of the graph of \( D'(t) \), the derivative of \( D(t) \). The function \( D'(t) \) is

- constant for \( 3 \leq t \leq 5 \), for \( 7 \leq t \leq 8 \), and for \( t \geq 10 \), and is
- linear for \( 5 \leq t \leq 7 \) and for \( 8 \leq t \leq 10 \).

![Graph of \( D'(t) \)]

a. [2 points] On which of the following intervals of \( t \) is the amount of user data stored on the server increasing for the entire interval? Give your answer as a list of one or more intervals, or write none.

\[ (0.5, 1.5), (1, 2), (7, 8), (10, 12) \]

**Solution:** (The amount of user data is increasing when its derivative, the function \( D'(t) \) graphed above, is positive.)

b. [2 points] When the amount of user data on the server is changing faster than 2 GB/hr, either increasing or decreasing, the server is said to be in an “excited state.” How many hours, between noon and midnight, does the server spend in an excited state?

**Solution:** The server spends 5 hours in an excited state. (This question is asking for the times \( t \) when \( D'(t) > +2 \) or \( D'(t) < -2 \). The graph lies above the line \( y = +2 \) for \( 6 < t < 8.5 \) and lies below the line \( y = -2 \) for \( 6 < t < 8.5 \).)

c. [2 points] The server hibernates when the amount of user data is not changing. How many hours, between noon and midnight, does the server spend in hibernation?

**Solution:** The amount of user data \( D(t) \) is not changing exactly when the derivative \( D'(t) \) equals zero. The total time in hibernation is therefore \( 5 - 3 = 2 \) hours.

d. [2 points] At midnight, 450 GB of data is stored on the server. If the rate of change of user data stays the same from midnight to 5 am the following morning, how much user data will be stored on the server at 5 am?

**Solution:** The graph above shows that \( D'(12) = -4 \), so at midnight, the amount of data is changing at a rate of \(-4 \) GB/hr. From midnight to 5 am the total change in the amount of data is therefore \( 5 \) hr \( \cdot \) \( -4 \) GB/hr = \(-20 \) GB. Since the server has 450 GB of data at midnight, there must be \( 450 - 20 = 430 \) GB of data at 5 am.