

4. [9 points]

Consider the piecewise function

$$q(x) = \begin{cases} 7e^{x-C} + \frac{x}{3x-2} & x < 0 \\ \frac{6+5x}{2+3x+4^x} & x \geq 0 \end{cases}$$

where C is a constant.

- a. [5 points] Find an exact value of C for which the function $q(x)$ is continuous at $x = 0$, or write NONE if there is no such value. Show your work and make sure your final answer is clear.

Solution: Set both pieces of the function equal for $x = 0$ and solve for C :

$$7e^{0-C} + \frac{0}{3(0)-2} = \frac{6+5(0)}{2+3(0)+4^0}$$

$$7e^{-C} = 2$$

(Below is one possible approach to solving for C .)

$$\ln(7e^{-C}) = \ln(2)$$

$$\ln(7) + \ln(e^{-C}) = \ln(2)$$

$$\ln(7) - C = \ln(2)$$

$$C = \ln(7) - \ln(2)$$

Answer: $C = \underline{\ln(7) - \ln(2)}$

Evaluate each of the expressions below. If a limit does not exist, including if it diverges to $\pm\infty$, write DNE. You do not need to show work.

- b. [2 points] $\lim_{x \rightarrow \infty} q(x)$

Solution:

$$\lim_{x \rightarrow \infty} \frac{6+5x}{2+3x+4^x} = 0$$

Answer: $\underline{0}$

- c. [2 points] $\lim_{x \rightarrow -\infty} q(x)$

Solution:

$$\lim_{x \rightarrow -\infty} \left(7e^{x-C} + \frac{x}{3x-2} \right) = \frac{1}{3}$$

Answer: $\underline{1/3}$