3. [5 points] The function \( y(t) \), given to the right, gives the mass, in milligrams, of a yeast colony \( t \) hours after an experiment begins, where \( A \), \( B \), and \( C \) are constants.

\[
y(t) = \begin{cases} 
  A & 0 \leq t < 4 \\
  12B^t & t \geq 4 
\end{cases}
\]

Find the values of \( A \), \( B \), and \( C \) such that all of the following hold:

- \( \lim_{t \to 0^+} y(t) = 8 \),
- the yeast colony’s mass decays by 2% each hour after \( t = 4 \), and
- \( y(t) \) is continuous at \( t = 4 \).

Show your work, and give your answers in exact form.

**Solution:** Because \( \frac{A}{1 + e^{-Ct}} \) is a continuous function, the first point gives

\[
8 = \lim_{t \to 0^+} \frac{A}{1 + e^{-Ct}} = \frac{A}{1 + e^{-C \cdot 0}} = \frac{A}{1 + 1},
\]

so \( A = 16 \). The second point gives \( B = 0.98 \).

Finally, the third point means we must have \( \lim_{t \to 4^-} y(t) = \lim_{t \to 4^+} y(t) = y(4) \). So we must have

\[
\frac{16}{1 + e^{-C \cdot 4}} = 12 \cdot (0.98)^4 \quad \text{so} \quad \frac{16}{12 \cdot (0.98)^4} - 1 = e^{-C \cdot 4}.
\]

Taking \( \ln \) of both sides and then dividing by \(-4\), we get \( C = -\frac{1}{4} \ln \left( \frac{16}{12 \cdot (0.98)^4} - 1 \right) \).

**Answers:** \( A = 16 \) \( B = 0.98 \) \( C = -\frac{1}{4} \ln \left( \frac{16}{12 \cdot (0.98)^4} - 1 \right) \)

4. [7 points] Consider the rational function \( r(x) = \frac{x(x - 1)(x + 4)}{(x^2 - 1)(x + 4)} \).

a. [5 points]

i. Find the equations of any horizontal asymptotes of \( r(x) \) or write NONE if there are none.

**Answer:** NONE

ii. Find all the zeros of \( r(x) \), or write NONE if there are none.

**Answer:** 0

iii. Find all numbers \( c \) such that the limit \( \lim_{x \to c} r(x) \) exists but \( r(c) \) is not defined.

**Answer:** \(-4, 1\)

b. [2 points] Find a linear function \( h(x) \) such that the function \( r(x) \cdot h(x) \) has no vertical asymptotes.

**Answer:** \( h(x) = x + 1 \)