3. [5 points] The function $y(t)$, given to the right, gives the mass, in milligrams, of a yeast colony t hours after an experiment begins, where A, B, and C are constants.

$$y(t) = \begin{cases} \dfrac{A}{1+e^{-Ct}} & 0 \le t < 4 \\ 12B^t & t \ge 4 \end{cases}$$

Find the values of A, B, and C such that all of the following hold:

- $\lim_{t\to 0^+} y(t) = 8$,
- the yeast colony's mass decays by 2% each hour after $t = 4$, and
- $y(t)$ is continuous at $t = 4$.

Show your work, and give your answers in **exact form**.

Solution: Because $\frac{A}{1+e^{-Ct}}$ is a continuous function, the first point gives

$$8 = \lim_{t\to 0^+} \frac{A}{1+e^{-Ct}} = \frac{A}{1+e^{-C\cdot 0}} = \frac{A}{1+1},$$

so $A = 16$. The second point gives $B = 0.98$.
Finally, the third point means we must have $\lim_{t\to 4^-} y(t) = \lim_{t\to 4^+} y(t) = y(4)$. So we must have

$$\frac{16}{1+e^{-C\cdot 4}} = 12\cdot(0.98)^4 \quad \text{so} \quad \frac{16}{12\cdot(0.98)^4} - 1 = e^{-C\cdot 4}.$$

Taking ln of both sides and then dividing by -4, we get $C = -\frac{1}{4}\ln\left(\frac{16}{12\cdot(0.98)^4} - 1\right)$.

Answers: $A =$ 16 $B =$ 0.98 $C =$ $-\dfrac{1}{4}\ln\left(\frac{16}{12\cdot(0.98)^4} - 1\right)$

4. [7 points] Consider the rational function $r(x) = \dfrac{x(x-1)(x+4)^2}{(x^2-1)(x+4)}$.

a. [5 points]

i. Find the equations of any horizontal asymptotes of $r(x)$ or write NONE if there are none.

Answer: **NONE**

ii. Find all the zeros of $r(x)$, or write NONE if there are none.

Answer: 0

iii. Find all numbers c such that the limit $\lim_{x\to c} r(x)$ exists but $r(c)$ is *not* defined.

Answer: $-4, 1$

b. [2 points] Find a linear function $h(x)$ such that the function $r(x)\cdot h(x)$ has no vertical asymptotes.

Answer: $h(x) =$ $x + 1$