3. [4 points] Find positive constants \( a, b, \) and \( c \) such that the function

\[
f(x) = \begin{cases} 
\ln(c \cdot e - x) & x \leq 0 \\
ax^3 + \pi & x > 0 
\end{cases}
\]

is continuous and satisfies \( \lim_{x \to \infty} f(x) = 4 \). Show your work, and write your answers in exact form.

**Solution:** If \( b > 3 \) then \( \lim_{x \to \infty} f(x) = 0 \), and if \( b < 3 \) then \( \lim_{x \to \infty} f(x) = \infty \); therefore, since \( \lim_{x \to \infty} f(x) = 4 \), we know \( b = 3 \). Then, since \( b = 3 \), we know that

\[
4 = \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{ax^3 + \pi}{x^3 + 1} = a.
\]

Finally, since \( f(x) \) is continuous at 0, the two parts of the piecewise definition of \( f(x) \) must agree at \( x = 0 \). Plugging \( x = 0 \) into both pieces, we get

\[
\ln(c e) = \ln(c) + 1 = \pi,
\]

so \( c = e^{\pi - 1} \).

**Answers:** \[a = 4, \quad b = 3, \quad c = e^{\pi - 1}\]

4. [5 points] Let

\[
Q(w) = w^w + \cos(6w - 1).
\]

Use the limit definition of the derivative to write an explicit expression for \( Q'(3) \). Your answer should not involve the letter \( Q \). Do not attempt to evaluate or simplify the limit. Write your final answer in the answer box provided below.

**Solution:**

\[
Q'(3) = \lim_{h \to 0} \frac{Q(3 + h) - Q(3)}{h} = \lim_{h \to 0} \frac{(3 + h)^3 + \cos(6(3 + h) - 1) - [3^3 + \cos(6 \cdot 3 - 1)]}{h}
\]

**Answer:** \[Q'(3) = \lim_{h \to 0} \frac{(3 + h)^3 + \cos(6(3 + h) - 1) - [3^3 + \cos(6 \cdot 3 - 1)]}{h}\]