3. [4 points] Find positive constants a, b, and c such that the function

$$f(x) = \begin{cases} \ln(ce - x) & x \leq 0 \\ \dfrac{ax^3 + \pi}{x^b + 1} & x > 0 \end{cases}$$

is continuous and satisfies $\lim_{x \to \infty} f(x) = 4$. Show your work, and write your answers in exact form.

Solution: If $b > 3$ then $\lim_{x\to\infty} f(x) = 0$, and if $b < 3$ then $\lim_{x\to\infty} f(x) = \infty$; therefore, since $\lim_{x\to\infty} f(x) = 4$, we know $b = 3$. Then, since $b = 3$, we know that

$$4 = \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{ax^3 + \pi}{x^3 + 1} = a.$$

Finally, since $f(x)$ is continuous at 0, the two parts of the piecewise definition of $f(x)$ must agree at $x = 0$. Plugging $x = 0$ into both pieces, we get

$$\ln(ce) = \ln(c) + 1 = \pi,$$

so $c = e^{\pi - 1}$.

Answers: $a =$ 4 $b =$ 3 $c =$ $e^{\pi-1}$

4. [5 points] Let

$$Q(w) = w^w + \cos(6w - 1).$$

Use the limit definition of the derivative to write an explicit expression for $Q'(3)$. *Your answer should not involve the letter Q. Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Solution:

$$\begin{aligned} Q'(3) &= \lim_{h\to 0} \frac{Q(3+h) - Q(3)}{h} \\ &= \lim_{h\to 0} \frac{(3+h)^{3+h} + \cos\big(6(3+h) - 1\big) - \big[3^3 + \cos(6\cdot 3 - 1)\big]}{h} \end{aligned}$$

Answer: $Q'(3) = \lim_{h\to 0} \dfrac{(3+h)^{3+h} + \cos\big(6(3+h) - 1\big) - \big[3^3 + \cos(6\cdot 3 - 1)\big]}{h}$