5. [7 points] Since the start of spring, bird enthusiasts Charlie and Parker have been seeing, and feeding, more and more birds in their backyard. Eventually they decide to model the number of birds they see and the amount of birdseed they go through using increasing functions.

Let $B(t)$ be the number of birds they see on the t^{th} day after the start of spring. They start recording values on the third day of spring, and their initial data appear in the table to the right.

t	3	4
$B(t)$	16	20

a. [1 point] Charlie thinks $B(t)$ should be a linear function. Find an expression for $B(t)$ if it is a linear function.

Answer: $B(t) =$ $4(t-3)+16$, **or** $4t+4$

b. [2 points] Parker thinks $B(t)$ should be exponential. Find an expression for $B(t)$ if it is an exponential function.

Solution: If $B(t)$ is exponential, then it has the form $B(t) = B_0a^t$. Thus $16 = B_0a^3$ and $20 = B_0a^4$. Dividing these equations gives

$$a = \frac{20}{16} = 5/4, \qquad \text{so} \qquad B_0 = \frac{16}{(5/4)^3}.$$

Answer: $B(t) = \dfrac{16}{(5/4)^3} \cdot (5/4)^t$

After arguing whether $B(t)$ is linear or exponential for (arguably) too long, and wasting a day in the process, they realize that taking a third measurement might settle the debate.

c. [2 points] Based on the additional data shown to the left, circle the one best answer:

t	3	4	6
$B(t)$	16	20	25

(i) $B(t)$ is linear but not exponential

(ii) $B(t)$ is exponential but not linear

(iii) $B(t)$ is both linear and exponential

(iv) $B(t)$ is neither linear nor exponential

Now let $S(t)$ be the amount of birdseed Charlie and Parker use, in pounds, on the t^{th} day after the start of spring.

d. [2 points] Write an equation involving B, S, and/or their inverses that represents the following statement:

Charlie and Parker use 2.3 pounds of birdseed two days after they see 43 birds.

Answer: $S^{-1}(2.3) = B^{-1}(43) + 2$