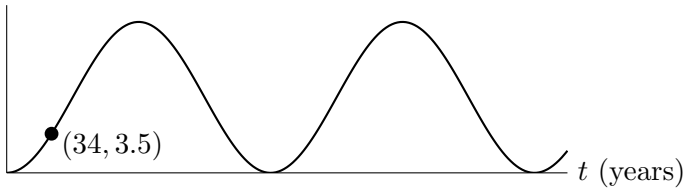
8. [8 points] Far in the future, a particular spaceship transports humans back and forth along a fixed path between the two star systems Alpha and Beta. The distance d in light-years from Alpha to the spaceship t years after its initial departure from Alpha is given by

$$d \;=\; s(t) \;=\; 6.75 - 6.75\cos\left(\tfrac{2\pi}{200}t\right).$$

A graph of $s(t)$, along with a table of a few values of both $s(t)$ and $s'(t)$, is given below.



t	0	9	12	34
$s(t)$	0	0.27	0.47	3.50
$s'(t)$	0	0.06	??	0.19

a. [1 point] How many years does it take for the spaceship to travel from Alpha to Beta?

Answer: **100** years.

b. [1 point] How many light-years apart are Alpha and Beta?

Answer: **13.5** light-years.

c. [1 point] Using the table, give the best possible estimate of $s'(12)$.

Answer: $\dfrac{0.47 - 0.27}{12 - 9}$ light-years per year.

There is a "Mystery Spot" along the spaceship's path where gravity seems to be a little different. The ship first passes this spot 34 years after departing Alpha. (See the graph above.)

d. [3 points] A baby tortoise is born on the spaceship just as it departs Beta, and becomes the ship's mascot, remaining aboard for the rest of its life. Assuming the tortoise lives 212 years, how many times will it get to see the Mystery Spot, and at what age(s)?

Solution: Using graph symmetry and the fact that the period of $s(t)$ is 200, the next three times after $t = 34$ that the ship passes by the Mystery Spot are $t = 166$, $t = 234$, and $t = 366$. Since the ship is at Beta at time $t = 100$, this means the ship passes by the Mystery Spot 66 years, 134 years, and 266 years after being at Beta. But the tortoise only lives 212 years, so it will only see the Mystery Spot twice, at ages 66 and 134.

Answer: **two** times, at ages 66 and 134.

e. [2 points] Now suppose the Mystery Spot wrecks the ship's engines at time $t = 34$, so the spaceship is left to drift along forever at the speed and in direction it was going when it reached the Mystery Spot. Under this new assumption, how far away would the spaceship be from Alpha two years after it reached the Mystery Spot? *(Include units. Note that $t = 34$ appears in the graph and in the table.)*

Solution: At time $t = 34$, the spaceship is 3.5 light-years from Alpha and moving directly away from Alpha at a speed of 0.19 light-years per year. So if it keeps moving away from Alpha at this same speed, in two years it will be $2 \cdot 0.19 = 0.38$ light-years farther away, so its total distance from Alpha will be $3.5 + 0.38 = 3.88$ light-years.

Answer: $3.50 + 2 \cdot 0.19 = 3.88$ **light-years**