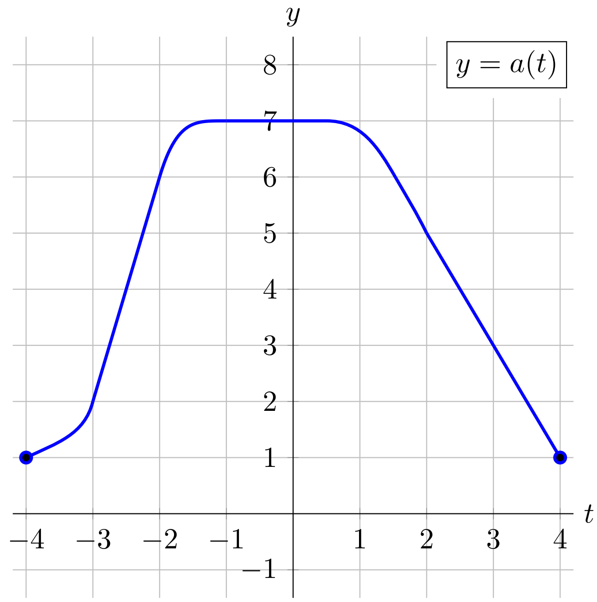
7. [5 points] Suppose $a(t)$ is the altitude in hundreds of meters above sea level of a certain hot air balloon t hours after 12pm noon on a sunny day. Carefully draw a plausible graph of $a(t)$ on the given axes, assuming the following are true:

- the balloon lifts off the ground at 8am from a point that is 100 meters above sea level, and stays in the air until it lands at the same location at 4pm;
- the rate at which the balloon's altitude is changing is constant between 9am and 10am, and again between 2pm and 4pm;
- at 9:30am, the balloon is **ascending** twice as fast as it is **descending** at 3pm;
- the balloon spends at least one full hour at its maximum altitude of 700 meters.



8. [6 points] Let $g(x)$ be the piecewise function defined by

$$g(x) = \begin{cases} \dfrac{-4(x+1)}{(x^2-1)(x+4)} & x < 0 \\ e^{A(x-1)} + \dfrac{B(x+1)^2(x-2)}{2(x-3)(x-2)^2} & x \geq 0 \end{cases}$$

where A and B are nonzero constants.

a. [3 points] List the x-coordinates of all **vertical asymptotes** of $g(x)$.

Answer: $x =$ $-4, 2, 3$

b. [3 points] Find values of the constants A and B such that $g(x)$ is continuous at $x = 0$ and $g(x)$ has a **horizontal asymptote** at $y = -3$.

Solution: We can get the horizontal asymptote at $y = -3$ if $A \leq 0$ and then

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{Bx^3}{2x^3} = B/2 = -3,$$

So $B = -6$. To make g continuous at 0, we set up the equation

$$\frac{-4(0+1)}{(0^2-1)(0+4)} = e^{A(0-1)} + \frac{-6(0+1)^2(0-2)}{2(0-3)(0-2)^2}, \quad \text{or} \quad 1 = e^{-A} - \frac{1}{2}.$$

Now solving for A by taking logarithms, we get

$$A = \ln(2/3).$$

Answer: $A =$ $\ln(2/3)$ and $B =$ -6