

5. [12 points] Abby is trying to finish writing an essay at a cafe while drinking coffee continuously. At this cafe, Abby only drinks the coffee that costs \$3 per cup, and the cafe does not sell fractional cups. Let $T(z)$ be the number of hours Abby has worked when she has consumed z milligrams of caffeine. Let $C(z)$ be the number of cups of coffee Abby needs to purchase to consume z milligrams of caffeine. Suppose T and T^{-1} are each both invertible and differentiable.

a. [2 points] The function C is not invertible. Explain why, using two sentences or fewer.

b. [3 points] Write a single **equation** involving T , T^{-1} and/or C that represents the following statement:

Abby has spent \$15 on coffee after working at the cafe for 4 hours.

Answer: _____

c. [3 points] Write a single **equation** involving T' , $(T^{-1})'$ and/or C' that represents the following statement:

After 0.05 hours of working on her essay at the cafe, Abby has consumed approximately 1 milligram of caffeine.

Answer: _____

5. (continued) The setup of the problem is restated here for your convenience.

Abby is trying to finish writing an essay at a cafe while drinking coffee continuously. At this cafe, Abby only drinks the coffee that costs \$3 per cup, and the cafe does not sell fractional cups. Let $T(z)$ be the number of hours Abby has worked when she has consumed z milligrams of caffeine. Let $C(z)$ be the number of cups of coffee Abby needs to purchase to consume z milligrams of caffeine. Suppose T and T^{-1} are each both invertible and differentiable.

d. [2 points] Suppose $T(95) = 3/2$ and $T'(95) = 1/60$. Give a formula for $L(z)$, the local linearization of T near $z = 95$.

Answer: $L(z) = \underline{\hspace{10cm}}$

e. [2 points] Use your linear approximation from the previous part to estimate how many milligrams of caffeine Abby has consumed after 1 hour and 36 minutes. Note that there are 60 minutes in one hour. You must show work supporting your final answer.

Answer: $\underline{\hspace{10cm}}$ milligrams