5. [12 points] Abby is trying to finish writing an essay at a cafe while drinking coffee continuously. At this cafe, Abby only drinks the coffee that costs \$3 per cup, and the cafe does not sell fractional cups. Let $T(z)$ be the number of hours Abby has worked when she has consumed z milligrams of caffeine. Let $C(z)$ be the number of cups of coffee Abby needs to purchase to consume z milligrams of caffeine. Suppose T and T^{-1} are each both invertible and differentiable.

a. [2 points] The function C is not invertible. Explain why, using two sentences or fewer.

Solution: If a cup of coffee has 100 milligrams of caffeine, the we have $C(1) = C(2)$. Thus, C must not be invertible.

b. [3 points] Write a single **equation** involving T, T^{-1} and/or C that represents the following statement:

Abby has spent \$15 on coffee after working at the cafe for 4 hours.

Answer: $3C(T^{-1}(4)) = 15$

c. [3 points] Write a single **equation** involving T', $(T^{-1})'$ and/or C' that represents the following statement:

After 0.05 hours of working on her essay at the cafe, Abby has consumed approximately 1 milligram of caffeine.

Answer: $(T^{-1})'(0.05) = 20$**,** $(T^{-1})'(0) = 20$**,** $T'(0) = 0.05$**, or** $T'(1) = 0.05$

5. (continued) The setup of the problem is restated here for your convenience.

Abby is trying to finish writing an essay at a cafe while drinking coffee continuously. At this cafe, Abby only drinks the coffee that costs \$3 per cup, and the cafe does not sell fractional cups. Let $T(z)$ be the number of hours Abby has worked when she has consumed z milligrams of caffeine. Let $C(z)$ be the number of cups of coffee Abby needs to purchase to consume z milligrams of caffeine. Suppose T and T^{-1} are each both invertible and differentiable.

d. [2 points] Suppose $T(95) = 3/2$ and $T'(95) = 1/60$. Give a formula for $L(z)$, the local linearization of T near $z = 95$.

Answer: $L(z) =$ $\frac{1}{60}(z - 95) + \frac{3}{2}$

e. [2 points] Use your linear approximation from the previous part to estimate how many milligrams of caffeine Abby has consumed after 1 hour and 36 minutes. Note that there are 60 minutes in one hour. You must show work supporting your final answer.

Solution: We set $L(z) = 1.6$ Then,

$$\frac{1}{60}(z - 95) + 1.5 = 1.6$$

$$\frac{1}{60}(z - 95) = .1$$

$$z - 95 = 6$$

$$z = 101.$$

Answer: 101 milligrams