6. [9 points] In this problem, we consider the function $f(x) = 4xe^{-x^2/8}$. In case it is helpful, recall that $e \approx 2.7$. The first derivative of $f(x)$ is given by:

$$f'(x) = (4 - x^2)e^{-x^2/8}.$$

For each part of this problem, **be sure you show enough evidence** to support your conclusions.

a. [5 points] Find the x-coordinates of all critical points of $f(x)$ on $(-\infty, \infty)$. Find the x-coordinates of the local maxima and local minima of $f(x)$, or write NONE in the appropriate blank if there are none of the specified type.

Solution: The critical points of $f(x)$ are the values where $f'(x)$ is zero or undefined. Here, $f'(x)$ is always defined. $f'(x) = 0$ when $(4 - x)^2 = (2 - x)(2 + x) = 0$. The critical points are $x = -2$ and $x = 2$.
To find the local max(es) and min(s), we apply the first derivative test to the critical points we found. We have $f'(x) = (2 - x)(2 + x)e^{-x^2/8}$. We then get the following sign chart:

$f'(x)$: $+ \cdot - \cdot + = -$ | -2 | $- \cdot - \cdot + = +$ | 2 | $- \cdot + \cdot + = -$

From this, we see that $x = -2$ is a local minimum and $x = 2$ is a local maximum.

Answer: Critical point(s): $x =$ ± 2

Answer: Local Max(es): $x =$ 2 Local Min(s): $x =$ -2

b. [4 points] Find the global maximum and minimum of $f(x)$ on $[1, \infty)$, or write NONE if there is no global extremum of that type. Give both the x-value(s) where $f(x)$ achieves the global max/min and the value of $f(x)$ at that x-value.

Solution: By part (a), we have that $x = 2$ is the only critical point on the interval $[1, \infty)$. Since 2 is a local maximum, we have that $f(x)$ has a global maximumum of $4(2)e^{-\frac{2^2}{8}} = 8e^{-1/2}$. We have $f(1) = 4e^{-1/8} > 0$ and $\lim_{x\to\infty} 4xe^{-x^2/8} = 0$. Thus, $f(x)$ does not have a global minimum on this interval.

Answer: $f(x)$ has a global max value of $y =$ $8e^{-1/2}$ occurring at $x =$ 2

Answer: $f(x)$ has a global min value of $y =$ **NONE** occurring at $x =$ **NONE**