7. [10 points] Given below is a table of values for a **continuous** function $q(t)$ and its first and second derivative. The question marks indicate values that are not known. For each question mark, the value may or may not exist, but at all other real numbers t, the values of $q(t)$, $q'(t)$, and $q''(t)$ exist.

t	-5	-3	-1	1	3	5	7	9
$q(t)$	1	?	0	?	?	?	?	?
$q'(t)$	?	0	-3	0	1	2	?	-3
$q''(t)$	?	-5	0	2	0	1	?	2

Between any two consecutive values of t in the table, both $q'(t)$ and $q''(t)$ are either **always positive** or **always negative**.

a. [2 points] Estimate $q'''(-2)$. Show your work to justify your answer.

Solution:

$$q'''(-2) \approx \frac{q''(-1) - q''(-3)}{-1-(-3)} = \frac{0-(-5)}{2} = 2.5$$

Answer: $q'''(-2) \approx$ 2.5

b. [1 point] Is $t = -3$ a local minimum of $q(t)$, a local maximum of $q(t)$, or is there not enough information to decide (NEI)? Circle **one** answer only. (No justification needed.)

LOCAL MIN   [LOCAL MAX]   NEI

Solution: Because $q''(-3) < 0$, the second derivative test tells us that $t = -3$ must be a local maximum as q is concave down there.

c. [2 points] Which of the following values of t must be inflection points of $q(t)$? Circle **all** correct answers. (No justification needed.)

$t = -3$ [$t = -1$] $t = 1$ $t = 3$ NONE OF THESE

d. [2 points] Which of the following values must be undefined? Circle **all** correct answers. (No justification needed.)

$q'(-5)$ [$q'(7)$] $q''(-5)$ [$q''(7)$]   NONE OF THESE

e. [3 points] Find the t-coordinate(s) of the **global minimum(s)** of $q(t)$ on the interval $[-5, 5]$, or write NEI if there is not enough information to determine this. Then, **briefly justify** your answer. You may give a possible sketch of $q(t)$ on $[-5, 5]$ as your justification.

Answer: Global min(s) at $t =$ 1

Justification:

Solution: The global minimum of $q(t)$ must be at $t = 1$. Because $t = -1$ and $t = 1$ are the only critical points of q inside this interval, and because the interval is closed and q is continuous (since it is differentiable), the global max(es) must occur at one of these points or an endpoint. We can see that $t = -5$ isn't a global min given the two known values of q. Also, $t = -3$ was a local max as determined above. Also, note that $q' > 0$ for $t > 1$, so q increases after this point. So the lowest value must occur at $t = 1$.