7.) The populations of Michigan and Arizona between the years of 1960 and 1990 can be modeled by the following functions, where t is the number of years since 1960, and the units of the population is in millions.

Michigan: $f(t) = 7.8(1.0058)^t$; **Arizona:** $g(t) = 1.3(1.035)^t$

(a) (3 pts) [No sentence necessary.] Over the 30 year period, what was the annual percent growth rate for the population of Arizona?

3.5%

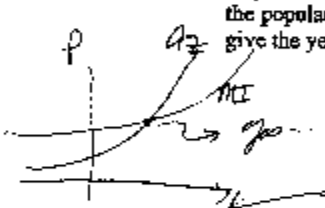
How much greater was that than the corresponding rate for Michigan?

$3.50 - 0.58 = 2.92\%$

(b) (2 pts) What was the difference in the two populations in 1960? [No sentence needed.]

$7.8 - 1.3 =$ 6.5 million people

(c) (4 pts) If the two states continue to grow according to the patterns given above, will there be a time when the population of Arizona will surpass that of Michigan? If not, explain (mathematically) why not. If so, give the year. [Show your work and express your answer in sentence form.]



$7.8(1.0058)^t = 1.3(1.035)^t \rightarrow \frac{\ln(7.8/1.3)}{\ln(1.035/1.0058)} = t \rightarrow t \approx 62.6$

$\frac{7.8}{1.3} = \left(\frac{1.035}{1.0058}\right)^t$

$1960 + 62.6 = 2022.6$

Yes, in the year 2022 the population of Arizona would surpass the population of Michigan.

(d) (2 pts) How many people would the model predict for the population of Michigan in the 2000 census? [No sentence necessary—show work.]

In 2000, $t = 40$,

$7.8(1.0058)^{40} \approx 9.83$ million people

(e) (2 pts) Interpret, in the context of this problem, the ***meaning*** of $g^{-1}(2)$. [Sentence form, of course.]

In this model, $g^{-1}(2)$ gives the year that the population of Arizona will have 2 million people.

(f) (3 pts) According to the model above, in what year was the population of Michigan 5 million people? [Show work and express answer in sentence form.]

We want $5 = 7.8(1.0058)^t \rightarrow \frac{5}{7.8} = (1.0058)^t$

so $t \cdot \ln(1.0058) = \ln\left(\frac{5}{7.8}\right) \rightarrow t = \frac{\ln(5/7.8)}{\ln(1.0058)} \approx -76.89$

$1960.00 - 76.89 = 1883.11$

According to this model, the population of Michigan was 5 million people in the year 1883!