

8.) **Essay Question.** All answers should be in complete sentences.

Average daily temperature for any city in the United States can be approximated with reasonable accuracy by a function of the form $f(t) = A \sin(b(t - h)) + k$, where t is in days after January 1.

For example, a model for average daily temps in the following cities is given by:

Phoenix, AZ: $f(t) = 20 \sin\left(\frac{2\pi}{365}(t - 109)\right) + 71$

Honolulu, HI: $f(t) = 4 \sin\left(\frac{2\pi}{365}(t - 141)\right) + 75$

Bismarck, ND: $f(t) = 30 \sin\left(\frac{2\pi}{365}(t - 110)\right) + 40$

- (a) (3 pts) Explain why it is appropriate to use $b = \left(\frac{2\pi}{365}\right)$.

Since $\frac{2\pi}{\text{period}} = b$ it makes sense for the period to be 365 days, $b = \frac{2\pi}{365}$.

The average temperature in Pittsburgh can be modeled by the function

$$f(t) = 22 \sin\left(\frac{2\pi}{365}(t - 118)\right) + 40$$

- (b) (3 pts) According to this model, what is the highest average temperature in Pittsburgh, and in approximately what month during the year does that occur?

The maximum is $40 + 22 = 62^\circ$, and this would occur @ $t = \frac{365}{4} + 118 = 209.25$ days after Jan 1. Thus, the average temperature is highest in late July in Pittsburgh.

- (c) (3 pts) What is the lowest average temperature in Pittsburgh, and in what month does that occur?

The minimum average temperature is $40 - 22 = 18^\circ$ and occurs when $t = \frac{365}{4} + 118 = 26.75$ days after Jan 1. Thus, the lowest average temperatures are in January and are $\approx 18^\circ$.

[This problem is continued on the next page.]

The model for average daily temperature from the previous page was given as

$$f(t) = A \sin(b(t - h)) + k.$$

(d) (3 pts) In this model, what does the parameter A tell you about the prevailing climate in a city?

The parameter A indicates the maximum number of degrees that the average temperature deviates (above or below) the mean temperature.

(e) (3 pts) What is the effect of the parameter h in the context of these models (i.e., in terms of temperature and days)?

The parameter h has the effect of moving the peak (highest temp) or valley (lowest temp) to different dates in the year.

(f) (3 pts) What does the parameter k indicate in terms of climate?

The constant k indicates the overall yearly average temperature -- or the median between the high & low temperatures.