8.) Essay Question. All answers should be in complete sentences.

Average daily temperature for any city in the United States can be approximated with reasonable accuracy by a function of the form $f(t) = A \sin(b(t - h)) + k$, where $t$ is in days after January 1.

For example, a model for average daily temps in the following cities is given by:

- Phoenix, AZ: $f(t) = 20 \sin \left( \frac{2\pi}{365} (t - 191) \right) + 71$
- Honolulu, HI: $f(t) = 4 \sin \left( \frac{2\pi}{365} (t - 141) \right) + 75$
- Bismarck, ND: $f(t) = 30 \sin \left( \frac{2\pi}{365} (t - 180) \right) + 40$

(a) (3 pts) Explain why it is appropriate to use $b = \frac{2\pi}{365}$.

Since $\frac{2\pi}{365}$ equals one cycle per year, it makes sense for $b$ to be 365 days, and $\frac{2\pi}{365}$ also.

The average temperature in Pittsburgh can be modeled by the function $f(t) = 22 \sin \left( \frac{2\pi}{365} (t - 118) \right) + 40$.

(b) (3 pts) According to this model, what is the highest average temperature in Pittsburgh, and in approximately what month during the year does that occur?

The maximum is $40 + 22 = 62{\circ}$, and this would occur around $\frac{2\pi}{365} \cdot 118 \approx 207.15$ days after Jan 1.

(c) (3 pts) What is the lowest average temperature in Pittsburgh, and in what month does that occur?

The minimum average temperature is $40 - 22 = 18{\circ}$ and occurs around $\frac{2\pi}{365} + 118 = 26.15$ days after Jan 1. Thus, the lowest average temperature is in January and is $18{\circ}$.

[This problem is continued on the next page.]
The model for average daily temperature from the previous page was given as

$$f(t) = A \sin(2\pi(t - h)) + k.$$ 

(d) (3 pts) In this model, what does the parameter $A$ tell you about the prevailing climate in a city?

The parameter $A$ indicates the maximum number of degrees that the average temperature deviates (above or below) the mean temperature.

(e) (3 pts) What is the effect of the parameter $h$ in the context of these models (i.e., in terms of temperature and days)?

The parameter $h$ shifts the effect of moving the peak (highest temp) or valley (lowest temp) to different dates in the year.

(f) (3 pts) What does the parameter $k$ indicate in terms of climate?

The constant $k$ indicates the overall yearly average temperature or the median between the high and low temperatures.