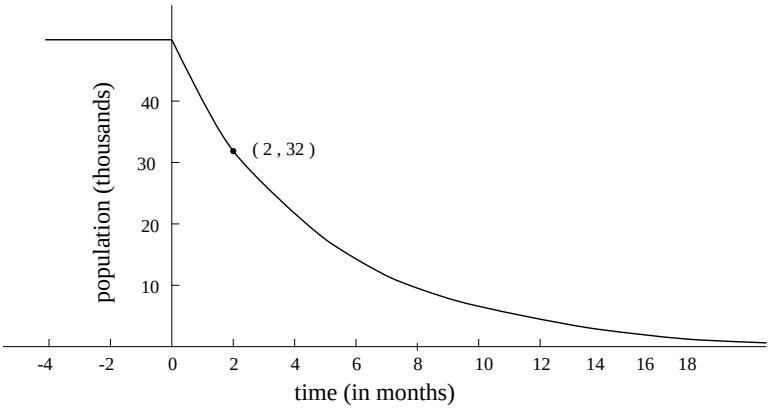
12. (12 points) For years the town of Hankytown has struggled with the city budget. The economy in Hankytown is based solely on the sale of valentines. Thus, the population in Hankytown varies each year from a high of 20,000 people (mostly migrant valentine makers) in February, to a low of 2,000 people in August.

On the other hand, Pankytown, just down the road from Hankytown, had a very stable economy. The population of 50,000 people had held steady for some time. However, in February of last year, the mayor of Pankytown decided to ban the purchase or sale of valentines. The citizens were outraged–rightfully so! People began to move as far away from Pankytown as they could get. The graph of $P(t)$ in the figure below shows what has happened to the population of Pankytown in the months since the ban took place.



(a) Determine a formula for an exponential function giving the population, P (in thousands), of Pankytown as a function of t, with $t = 0$ representing the first of February last year. [Note: the formula will only model the population for $t \geq 0$ in this picture.]

$P(t) =$ _____

(b) (i) On the same set of axes with $P(t)$ above, sketch a graph of the function which represents the population of Hankytown, H (in thousands), as a function of t, with $t = 0$ representing the first of February.
(ii) Determine a formula for a trigonometric function giving the population $H(t)$.

$H(t) =$ _____

(c) Is there a time (or times) that these models indicate that the populations of Pankytown and Hankytown are the same? If so, when? If not, explain why not.