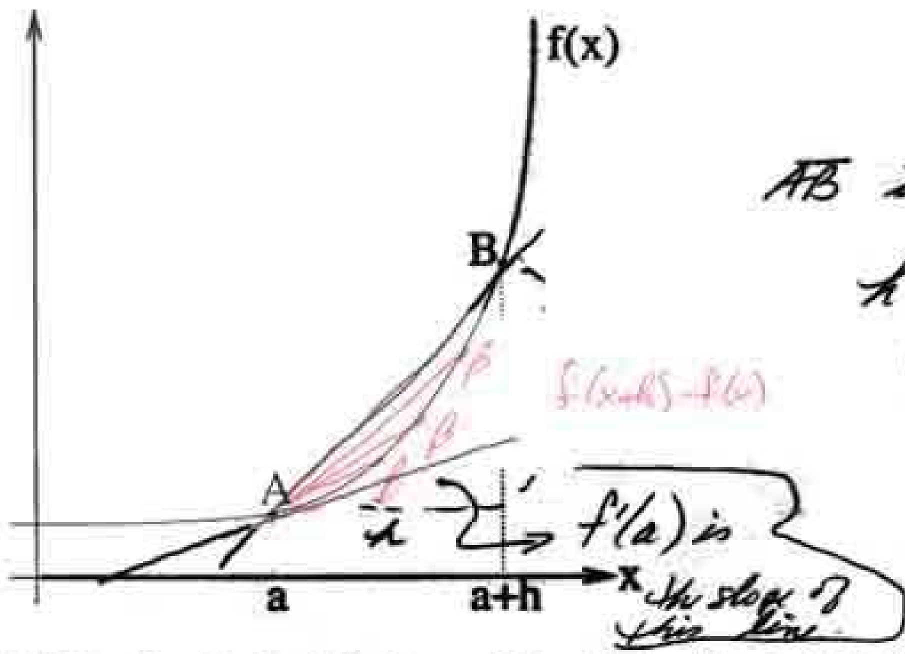
11. (11 pts.) (a) Give the limit definition of the derivative of a function f at a point a.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

(b) One interpretation of the derivative, $f'(a)$, is that it represents the slope of the tangent line to the graph of f at the point A. Use the limit definition of the derivative and the figure below to show why this interpretation is valid. Feel free to use the space to the right to explain your drawing.



The slope of the line AB is $\frac{f(a+h)-f(a)}{h}$. As $h \to 0$, then $B \to A$ and the slope of AB approaches the slope of the tangent to the curve at A.

(c) Use the limit definition of the derivative to find $g'(x)$ for the function $g(x) = 2x^2 - 3x$.

$$g'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$\lim_{h\to 0} \frac{2(x^2+2xh+h^2) - \cancel{3x} - 3h - 2x^2 + \cancel{3x}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 3h - \cancel{2x^2}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = 4x - 3$$