12. (14 pts.) Sunny and Tyrrell have been dating since New Year’s Eve. Sunny has noted that the amount of affection she has for Tyrrell, measured in bushels, is growing at a linear rate. However, since she is a math major, she tells her friend that her affection is growing as the slope of the line tangent to the curve $f(t) = \sqrt{t}$ at the point $(4, 2)$, where $t$ is in weeks since the first of January.

(a) At what rate is Sunny’s affection for Tyrrell growing? Write your answer in a complete sentence.

$$f(t) = \sqrt{t} \implies f'(t) = \frac{1}{2} t^{-\frac{1}{2}}$$

Sunny’s affection is growing at the rate $8 \frac{1}{2}$ bushels per week.

(b) Find an equation of the line that is tangent to $f$ at the point $(4, 2)$. This is the model for Sunny’s affection, $S(t)$.

$$S(t) = \frac{1}{4} t + 1$$

Tyrrell, being an applied mathematician, determines that he can model his affection for Sunny according to the power function $T(t) = k t^2$ (again in terms of bushels and weeks).

(c) If Tyrrell’s model passes through the point $(8, 3)$, what is $k$?

$$3 = k(8)^2 \implies k = \frac{3}{64}$$

(d) If Sunny and Tyrrell’s affection models continue to hold, and if the person with the most affection for the other buys Valentine flowers, who will buy the flowers? Explain. (Hint: Valentine’s day is two days from now.)

Valentine’s Day occurs on 2/14.

$$S(6) = \frac{1}{4} 6 + 1 = 2.5$$ bushels of affection.

$$T(6) = \frac{3}{64} (36) = 1.6875$$ bushels of affection.

Sunny will buy the flowers.

(e) Is there a time that Sunny and Tyrrell will have equal affection for one another? If so, approximately when. If not, why not?

Yes, Sunny and Tyrrell will have equal affection for one another at $t = 8$, or in early March. At that time each will have 3 bushels of affection for the other person.