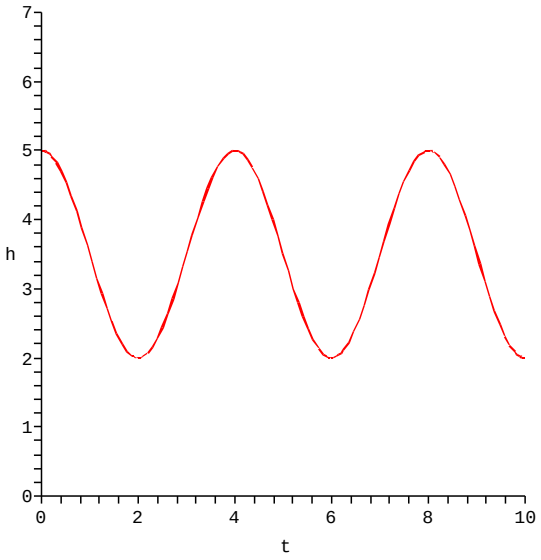
10. (4 points each) At a nearby elementary school the seats of the swing set sit 2 feet off the ground when at rest. While observing a child swing, you note that the seat reaches a maximum height of 5 feet from the ground when the child swings without the aid of pushing from an adult. It takes the child 4 seconds to travel between successive maximum heights. (One is achieved while swinging forward, one while swinging backwards.)

(a) Sketch a graph of the seat's height above the ground (in feet) as a function of time (in seconds) on the axes provided below. Assume that at $t = 0$ the child is at her maximum height, and that she reaches the same maximum height each swing through. Be sure to label the axes carefully! .



(b) Write a trigonometric equation describing the height of the seat as the child swings back and forth.

The highest point will be $h = 5$ and the lowest point will be $h = 2$, so the midline is given by $h = 3.5$. The amplitude is $A = \frac{5-2}{2} = \frac{3}{2}$. The period is 4, and so $B = \frac{\pi}{2}$. Therefore the equation is given as

$$h(t) = \frac{3}{2}\cos(\frac{\pi}{2}t) + 3.5.$$

(c) At which time(s) during the first 4 seconds of motion is the height of the seat changing most rapidly?

The times when the height of the seat is changing most rapidly occur when the function is the steepest, ie., when the absolute value of the derivative is at its largest. This occurs at $t = 1$ and $t = 3$ seconds during the first cycle.