

4. (6 points) A circus is planning to visit your hometown. They claim to have a mummy that is 15,000 years old, but the citizens of your town are suspicious. The town council is given a sample of the mummy for a carbon-14 analysis. Your old high school science teacher is able to find that 33% of the mummy's carbon-14 remains. Using the fact that the half-life of carbon-14 is 5,730 years, determine the age of the mummy that the circus is bringing to town. [Show your work!]

We know that since the carbon-14 decays exponentially, that the formula for the amount of carbon-14 left after t years is given by

$$P = P_0 e^{rt}$$

where P_0 is the initial amount and r is the exponential rate of decay.

To find r , we use that the half-life of carbon-14 is 5730 years. So we have

$$\begin{aligned} \frac{1}{2}P_0 &= P_0 e^{5730r} \\ \frac{1}{2} &= e^{5730r} \\ -\ln(2) &= 5730r \\ r &= \frac{-\ln(2)}{5730}. \end{aligned}$$

So now we can find the time elapsed when 33% of the mummy's carbon remains.

$$\begin{aligned} 0.33P_0 &= P_0 e^{\frac{-\ln(2)t}{5730}} \\ \ln(0.33) &= \frac{-\ln(2)t}{5730} \\ t &= 9,164.92 \text{ years} \end{aligned}$$

Therefore, the circus is exaggerating the age of their mummy.

5. (3 points each) The marketing department of Lay's Potato Chips decides to do a study on the number of chips a person craves as a function of the number of chips already eaten by that person. Their function turns out to be the rational function

$$C(x) = \frac{2x^2}{(x-1)^2},$$

where $C(x)$ is the number of additional chips a person who has eaten x chips is still craving to eat.

(a) How is Lay's famous slogan "You can't eat just one" summarized by this equation?

The slogan is given by the fact that 1 is not in the domain of $C(x)$. This shows that it is impossible to eat one Lay's potato chip.

(b) What is the horizontal asymptote of $C(x)$? What does this say about a person with an unlimited supply of chips who can't control his/her cravings?

The horizontal asymptote is at $y = 2$. This says that in the long run a person will always crave 2 more Lay's potato chips. So if one could not control one's cravings, one would eat potato chips forever.