6. (12 points) For this problem f is differentiable everywhere.

(a) Let $g(x) = f(x-2)$. Describe the graph of $g(x)$ in terms of the graph of $f(x)$.

The graph of g is the graph of f shifted to the right 2 units.

(b) If $f'(1) = 6$, what is $g'(3)$? Don't do any calculations here, use the geometry of the situation from part (a) to arrive at your answer.

Since we have just shifted the graph of f to the right by 2, we must have $g'(3) = f'(1) = 6$. So we are looking at the slope of the tangent line to f at $x = 1$, only shifted along with the graph of g.

(c) State the limit definition of the derivative for the function f.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(d) Let $j(x) = f(x) + 10$. Use the limit definition of the derivative to calculate the derivative of j in terms of the derivative of f.

$$\begin{aligned} j'(x) &= \lim_{h \to 0} \frac{j(x+h) - j(x)}{h} \\ &= \lim_{h \to 0} \frac{(f(x+h) + 10) - (f(x) + 10)}{h} \\ &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= f'(x). \end{aligned}$$