8. (3 points each) A study is published by a group of researchers at a prominent university that gives a person’s expected annual salary after 10 years of work as a function, $f$, of the total amount of money that that person spent on college tuition. (The group counts loans, scholarships, family contributions, etc., as tuition that a person pays.) The tuition and salary are both measured in thousands of dollars.

(a) What does the statement $f(5) = 20$ mean in practical terms?

The statement $f(5) = 20$ means that the expected annual salary after 10 years of work for a person who spent 5 thousand dollars on college tuition is 20 thousand dollars.

(b) What does $f^{-1}(50) = 20$ mean in practical terms?

The expression $f^{-1}(50) = 20$ tells us that if one wants to have an annual salary of 50 thousand dollars after 10 years of work, then one should spend 20 thousand dollars on college tuition.

(c) What do high-priced private schools hope is true about the sign of $f''$? Explain.

A high-priced private school would hope that the more one spends on college tuition, the higher the expected annual salary. Otherwise students would not be willing to pay more money to attend their school. So they would hope the function is an increasing function, i.e., that the derivative of $f$ is positive.

(d) What does the statement $f'(35) = 3$ mean in practical terms?

The expression $f'(35) = 3$ tells us that if a person spends 35 thousand on college, then the expected annual salary after 10 years of work would be approximately 3 thousand dollars more for an additional 1 thousand dollars spent on college tuition.

(e) Suppose you are trying to pick a college and your only concern is your expected salary after 10 years of work. If one of the schools you are considering will cost you 80,000 in tuition and $f'(80) = -0.5$, should you choose a more expensive, less expensive, or that particular school? Justify your answer.

The derivative being negative tells you that the function $f$ is decreasing when one spends $80$ thousand for college tuition. Therefore, you would be wise to attend a school that would cost you less than $80,000.